Derivatives Lab

Session 21

November 27, 2012

Suppose S_i for $i = 0, \ldots, N$ denotes time series data which we believe behaves like geometric Brownian motion. Then estimates for the parameters μ and σ can be obtained in the following way.

Consider the log-returns

$$r_i = \ln S_{i+1} - \ln S_i \,,$$

then compute the sample mean

$$\bar{r} = \frac{1}{N} \sum_{i=0}^{N-1} r_i$$

and sample variance

$$\sigma_r^2 = \frac{1}{N-1} \sum_{i=0}^{N-1} (r_i - \bar{r})^2 \,.$$

Then the estimates for σ and μ are given by

$$\hat{\sigma} = \frac{\sigma_r}{\sqrt{\Delta t}} \qquad \text{and} \qquad \hat{\mu} = \frac{\bar{r}}{\Delta t} + \frac{\hat{\sigma}^2}{2} \,.$$

1. Generate a large number of sample geometric Brownian paths with fixed μ and σ. For each, compute the estimates $\hat{\sigma}$ and $\hat{\mu}$.

    (a) Draw a histogram (command `hist`) for the distribution of $\hat{\sigma}$ and $\hat{\mu}$.

    (b) It is known that the variance of the estimate for σ is approximately

    $$\mathrm{Var}[\hat{\sigma}] = \frac{\hat{\sigma}^2}{2N} \,.$$

    Does your statistics from part (a) reproduce this result?

    (c) What is the variance of $\hat{\mu}$? Is it large or small?

2. Modify your code in such a way that the number of points on the geometric Brownian path is $N = 2^k + 1$ so that the number of log-returns is a power of two. Now repeat the estimation of σ for a single geometric path over large time steps of length $2^i \, \Delta t$ where $i = 0, \ldots, k-1$ and Δt is the time step of the original geometric Brownian path. Plot the $\hat{\sigma}$ vs. the log of the number of sample points (`semilogx`).

   Describe, in writing, how this result changes if

   (a) you add Gaussian noise to the geometric Brownian motion;

   (b) you add a high frequency periodic perturbation?

3. Perform a QQ-plot vs. the normal distribution for each of the three cases considered above.

   Discuss the result in writing.

4. Plot the autocorrelation function for each of the three cases considered above.

   Discuss the result in writing.