Applied Calculus

Homework 4

Due in class, October 13, 2015

1. (From MLS, p. 322.) Suppose that the population of a town can be approximated by

$$P(t) = 16\,250 \cdot (0.87)^t\,,$$

where P is the population of the town t years after 1985 (starting January 1, 1985).

   (a) Find the rate of change of the population on January 1, 2000. Is the population increasing or decreasing?

   (b) Find the rate of change of the population on January 1, 2010. Is the rate of change faster or slower than in was in 2000?

   (c) In this model, what happens for very long times (mathematically in the limit as $t \to \infty$?)

2. (From MLS, p. 322.) A laboratory study investigating the relationship between diet and weight in adult humans found that the weight of a subject, W (in pounds), was a function $W = f(c)$, of the average number of calories per day, c, consumed by the subject.

   (a) Interpret the statements $f(1800) = 155$, $f'(2000) = 0$ each in terms of diet and weight.

   (b) What are the units of $f'(c)$?

3. You are recording the lap times of a runner each of the four 400 m laps of a 1 600 m race. Is the average velocity of the runner over the entire race equal to the arithmetic mean of the average lap velocities computed from the lap times? Explain why or why not.

4. Find the equation of the tangent line at $x = 1$ for the following functions:

   (a) $f(x) = \dfrac{x \ln x}{x+1}$

   (b) $f(x) = \sqrt{1 + \mathrm{e}^x}$

5. For each function f: Find the vertical and horizontal asymptotes (if any), find and classify all critical points, determine where the function is concave up or concave down, find all points of inflection, and sketch the graph.

   (a) $f(x) = x^5 - 2\,x^3 + x - 2$

   (b) $f(x) = \dfrac{x+1}{x-1}$

   (c) $f(x) = \ln(x^2 + 1)$

   (d) $f(x) = x\,\mathrm{e}^{-x}$

   (5 Points each)

6. *Note:* The solution to this homework may be turned in *after* the midterm exam on Tuesday, October 20, in class.

   In *Scientific Python*, after setting up your environment with `%pylab inline`, generate a data set as follows

```
x = linspace(0,10,30)
y = 0.5*x + 0.1*normal(size=30)
```

   (a) Explain what these commands do and what you expect from this data set.

   (b) Compute the local rate of change in between each of the data points.

   Hint: You might find the partial data vectors `y[1:]` and `y[:-1]` useful. Find out what they are, and how to use them in the context of this question.

   (c) Compute the average rate of change using two different methods: (i) by doing a least-square fit of a straight line and looking at its slope, (ii) by computing the average rate of change from the local rates of change in part (b). Do the results coincide? Which computation is better? Explain.