Applied Calculus

Homework 6

Due in class, November 17, 2015

1. Find the minimum of the function $f(x, y) = x^2 - x + y + y^2$.

2. (From MLS, p. 406.) Find the rectangle with the largest area that can be placed inside the region bounded between the curve $y = 4 - x^2$ and the horizontal axis $y = 0$.

   (a) Solve this problem by formulating it as a maximization problem for a function of one variable.

   (b) Solve this problem by using a Lagrange multiplier.

3. Use a Lagrange multiplier to find the point on the unit circle $x^2 + y^2 = 1$ closest to $(1, 2)$.

4. Use Lagrange multipliers to find the maximum and minimum values of the function

$$f(x, y, z) = 3\,x - y + 2\,z$$

   subject to the constraints

$$x + y - z = 0 \qquad \text{and} \qquad x^2 + 3\,y^2 = 1\,.$$

5. Heron's formula states that the area of a triangle with side lengths x, y, and z is

$$A = \sqrt{s\,(s - x)\,(s - y)\,(s - z)}$$

   where s is the semiperimeter of the triangle, i.e.,

$$s = \frac{x + y + z}{2}\,.$$

   Use a Lagrange multiplier to show that the triangle with maximum area for a given perimeter is equilateral.