Recall initial tableau from last lecture

x_1	x_2	u	v	s_1	s_2	s_3	
1	1	-1	1	0	0	0	1
2	-1	-2	2	1	0	0	5
1	-1	0	0	0	1	0	4
0	1	1	-1	0	0	1	5
-1	-2	-3	3	0	0	0	0

after elimination, tracks value of $-z$ of current basic solution

$-2R1+R2 \to R2$, $-R1+R3 \to R3$, $R1+R5 \to R5$

x_1	x_2	u	v	s_1	s_2	s_3	
1	1	-1	1	0	0	0	1
0	-3	0	0	1	0	0	3
0	-2	1	-1	0	1	0	3
0	1	1	-1	0	0	1	5
0	-1	-4	4	0	0	0	1

largest negative entry in last row → entering variable

$\frac{3}{1}$ ✓

$\frac{5}{1}$ ✗

$x_1 = 1$, $s_1 = 3$, $s_2 = 3$, $s_3 = 5$

$x_2 = u = v = 0$

$z = -1$

Two options for a pivot in column u, R3 or R4.

Suppose we take R4 with pivot in column u: $R3 - R4 \to R3$

New row 3 would be: 0 -3 0 0 0 1 -1 -2

pivot

$s_2 = -2$ forbidden!

So choose pivot from row 3:

$R3 + R1 \to R1$

$R4 - R3 \to R4$

$4R3 + R5 \to R5$

x_1	x_2	u	v	s_1	s_2	s_3	
1	-1	0	0	0	1	0	4
0	-3	0	0	1	0	0	3
0	-2	1	-1	0	1	0	3
0	3	0	0	0	-1	1	2
0	-9	0	0	0	4	0	13

negative entry → x_2 is entering

only possible new pivot!

$x_1 = 4$, $u = 3$, $s_1 = 3$, $s_3 = 2$

$x_2 = v = s_2 = 0$

$z = -13$

$\frac{1}{3}R4 + R1 \to R1$

$R4 + R2 \to R2$

1	0	0	0	0	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{14}{3}$
0	0	0	0	1	-1	1	5

$x_1 = \frac{14}{3}$

$x_2 = \frac{2}{3}$

$v = s_2 = s_3 = 0$

$$
\begin{array}{l}
\frac{2}{3}R4 + R3 \to R3 \\
\frac{1}{3}R4 \to R4 \\
\\
3R4 + R5 \to R5
\end{array}
\qquad
\begin{array}{ccccccc|c}
0 & 0 & 1 & -1 & 0 & \frac{1}{3} & \frac{2}{3} & \frac{13}{3} \\
0 & 1 & 0 & 0 & 0 & -\frac{1}{3} & \frac{1}{3} & \frac{2}{3} \\
\hline
0 & 0 & 0 & 0 & 0 & 1 & 3 & 19
\end{array}
\qquad
\begin{array}{l}
u = \frac{13}{3} \\
s_1 = 5
\end{array}
\qquad
z = -19
$$

↑ all positive (or zero), no further improvement is possible.

<u>Pivoting rules:</u>

(i) As entering variable column, choose the column with the largest negative entry in the objective function row.

If all entries are non-negative, terminate.

(ii) As the new row with pivot, choose that row with the least positive ratio of right-hand coefficient to coefficient in that column.

If not possible because all coefficients in that column are negative, terminate with an unbounded feasible region and arbitrarily small objective function.