Towards the dual LP problem

"Car factory problem" (like WYNDOR):

- produce 2 products: cars (x_1) and trucks (x_2)

maximize $z = 3x_1 + 2x_2$

← profit per truck

↑ profit per car

subject to

$5x_1 \leq 100$ "car assembly"

100h available

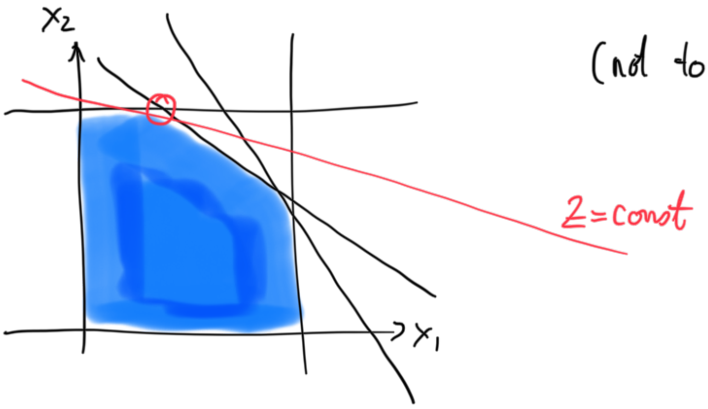
↑ one car needs 5h assembly

$10x_2 \leq 100$ "truck assembly"

$4x_1 + 3x_2 \leq 100$ "metal stamping process"

$3x_1 + 5x_2 \leq 100$ "engine assembly"

$x \geq 0$



(not to scale)

Via simplex method:

x_1	x_2	s_1	s_2	s_3	s_4	
5_{1}		$1_{\frac{1}{5}}$				100_{20}
	10		1			100
4_{0}	3_{3}	$-\frac{4}{5}$		1_{1}		100_{20}
3_{0}	5_{5}	$-\frac{3}{5}$			1_{1}	100_{40}

$5x_1 \leq 100$

$\Leftrightarrow 5x_1 + s_1 = 100$

-3_{0}	-2_{-2}	$\frac{3}{5}$				0_{60}	$R4 + 3R1 \to R4$

Now x_2 is new entering variable, new pivot is in third row

x_1	x_2	s_1	s_2	s_3	s_4		
1		$\frac{1}{5}$				20	
0	0	*	1	*	0	$100 - \frac{200}{3}$	
	1	$-\frac{4}{15}$		$\frac{1}{3}$		$\frac{20}{3}$	
	0	*		*	1	$40 - 5\,\frac{20}{3}$	
0	0	>0	0	>0	0	$60 + 2 \cdot \frac{20}{3}$	$= 73,\ldots$

Look at structure:

$$\left.\begin{array}{ll} n=2 & \text{physical variables} \\ m=4 & \text{constraints} \end{array}\right\} \quad n+m=6 \text{ variables altogether}$$

Unless there is some redundancy, there will be m basic variables.

Typically, all physical variables are basic (assume for simplicity), so we have $m-n$ slack variables that are basic.

Q: How many slack variables are non-basic? n!!

The non-basic slack variable point to the binding constraints

satisfied with equality

Typically: as many binding constraints as physical variables.

Crossing out non-binding constraints and non-basic slacks does not change the solution, so it can be written

$$\tilde{A} x = \tilde{b} \qquad \text{where } \tilde{A} = \begin{pmatrix} 5 & 0 \\ 4 & 3 \end{pmatrix}$$

$\tilde{\ }$ $/100/$

$$b = \begin{pmatrix} 100 \end{pmatrix}$$

are the remaining coefficients

$$\Rightarrow \qquad x = \tilde{A}^{-1} \tilde{b}$$

$$z = c^T x = c^T \tilde{A}^{-1} \tilde{b}$$