The newsvendor problem:

c: unit cost for product

h: holding cost

p: shortage cost

y: decision variable, # of unit to put in stock

D: random variable, demand

$$C = cy + p \max\{0, D-y\} + h \max\{0, y-D\}$$

$$\mathbb{E}[C] = \sum_{d=0}^{\infty} C(d)\, P_d$$

expected value

Goal: minimize $\mathbb{E}[C]$ as a function of y.

Solution strategy ①: Use empirical data → empirical probability distribution
→ brute-force optimization

Example: 20 days data

9, 15, 14, 9, 10, 11, 10, 7, 2, 7, 10, 11, 8, 20, 10, 10, 12, 13, 10, 9

# sold	2	7	8	9	10	11	12	13	14	15	20
# days	1	2	1	3	6	2	1	1	1	1	1
prop.	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{3}{10}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{1}{20}$	$\frac{1}{20}$	$\frac{1}{20}$	$\frac{1}{20}$

$$\mathbb{E}[C] = C(2)\tfrac{1}{20} + C(7)\tfrac{1}{10} + C(8)\tfrac{1}{20} + C(9)\tfrac{3}{20} + \cdots$$

Solution strategy 2:

Approximate P_d by a continuous probability distribution $\varphi(d)$

$$\mathbb{E}[C] \approx \int_0^{\infty} C(\xi)\, \varphi(\xi)\, d\xi$$

ξ: demand

$\varphi(\xi)$: probability density of demand

$$= \int_0^{\infty} \left[cy + p \max\{0, \xi - y\} + h \max\{0, y-\xi\} \right] \varphi(\xi)\, d\xi$$

$$= cy \underbrace{\int_0^{\infty} \varphi(\xi)\, d\xi} + p \int_y^{\infty} (\xi - y)\, \varphi(\xi)\, d\xi + h \int_0^{y} (y-\xi)\, \varphi(\xi)\, d\xi$$

1

$$\frac{d\mathbb{E}[C]}{dy} = c - p\,\underbrace{(y-y)}_{=0}\,\varphi(y) + p\,\underbrace{\int_y^\infty (-1)\,\varphi(\xi)\,d\xi}_{= \int_0^y \varphi(\xi)\,d\xi - \int_0^\infty \varphi(\xi)\,d\xi = \Phi(y) - 1} + h\,\underbrace{(y-y)}_{=0}\,\varphi(y) + h\,\underbrace{\int_0^y \varphi(\xi)\,d\xi}_{\Phi(y)}$$

$$\Phi(a) = \int_0^a \varphi(\xi)\,d\xi$$ probability that demand $\leq a$

"Cumulative distribution function" (CDF)

$$\Rightarrow \quad c + p\,(\Phi(y) - 1) + h\,\Phi(y) = 0$$

$$\Rightarrow \quad c - p = -\Phi(y)\,(p+h)$$

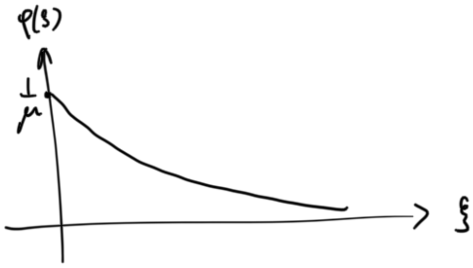
$$\Rightarrow \quad \boxed{\Phi(y) = \frac{p-c}{p+h}}$$

probability that demand is satisfied "optimal service level"

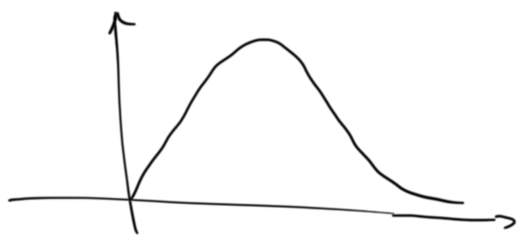
<u>Example:</u> Assume exponential distribution

$$\varphi(\xi) = \frac{1}{\mu}\, e^{-\frac{\xi}{\mu}}$$

μ : mean (average) number sold



more realistic



$$\Phi(a) = \int_0^a \frac{1}{\mu}\, e^{-\frac{\xi}{\mu}}\,d\xi = -e^{-\frac{\xi}{\mu}}\Big|_0^a = 1 - e^{-\frac{a}{\mu}}$$

$$\Phi(y) = \frac{p-c}{h+p} \qquad \Rightarrow \quad 1 - e^{-\frac{y}{\mu}} = \frac{p-c}{h+p}$$

$$\Rightarrow \quad 1 - \frac{p-c}{h+p} = e^{-\frac{y}{\mu}}$$

$$\Rightarrow \quad \frac{h + \cancel{p} - \cancel{p} - c}{} = e^{-\frac{y}{\mu}}$$

$$h+p$$

$$\Rightarrow -\frac{y}{\mu} = \ln \frac{h-c}{h+p}$$

$$\Rightarrow y = \mu \underbrace{\ln \frac{h+p}{h-c}}_{1.19}$$

Numerical example:

$c = 20$

$h = -9$

$p = 45$

So for 100 customers on average, should stock 119 items