Calculus and Elements of Linear Algebra I

Homework 1

Due on Moodle, September 14, 2020

1. (a) Find the (complex) roots of the polynomial

$$p(x) = x^2 + 4\,x + 13\,.$$

   (b) Find the values of the parameter λ for which the equation

$$2\,x^2 - \lambda\,x + \lambda = 0$$

   has no real solutions.

2. Find all the roots (real or complex) of the polynomial

$$p(x) = 24 - 8\,x - 18\,x^2 + 18\,x^3 - x^4 - 4\,x^5 + x^6\,.$$

   Hint: $x = 3$ is a root. Divide out the associated linear factor and continue with more roots that are easy to guess.

3. Assuming that $z = a + b\mathrm{i}$ is a complex number, compute real and imaginary parts of

   (a) $\dfrac{1}{z^2}$,

   (b) $\dfrac{z+1}{2z-5}$,

   (c) z^3.

4. (a) Compute $\left|\dfrac{1+\mathrm{i}}{2-\mathrm{i}}\right|$.

   (b) Characterize the set of real numbers x that satisfy

$$|6 - 4\,x| \geq |x - 2|\,.$$

5. (a) For $v, w \in \mathbb{C}$, show that $v^*\,w^* = (vw)^*$.

   (b) Conclude, inductively, that $(z^n)^* = (z^*)^n$ for $z \in \mathbb{C}$ and $n \in \mathbb{N}$.