Calculus and Elements of Linear Algebra I

Homework 11

Due on Moodle, Monday, November 30, 2020

1. Solve the following system of linear equations using the method taught in class.

$$\begin{aligned} x_1 + 3\,x_2 - 5\,x_3 &= 4 \\ x_1 + 4\,x_2 - 8\,x_3 &= 7 \\ -3\,x_1 - 7\,x_2 + 9\,x_3 &= -6 \end{aligned}$$

2. Find conditions on α such that following system of linear equations has (a) exactly one solution, (b) no solutions, or (c) an infinite number of solutions; give all solutions where they exist.

$$\begin{aligned} x_1 + \alpha\,x_2 &= 1 \\ x_1 - x_2 + 3\,x_3 &= -1 \\ 2\,x_1 - 2\,x_2 + \alpha\,x_3 &= -2 \end{aligned}$$

3. Let $\boldsymbol{v} = (1,2,3)^T$ be a vector expressed in coordinates with respect to the standard basis of $\mathbb{R}^3$. Find the coordinates of this vector with respect to the basis

$$\boldsymbol{b}_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \boldsymbol{b}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad \boldsymbol{b}_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$$

4. Determine whether the following vectors form a basis of $\mathbb{R}^4$. If not, obtain a basis by adding and/or removing vectors from the set.

$$\boldsymbol{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \boldsymbol{v}_2 = \begin{pmatrix} 1 \\ -1 \\ -1 \\ 2 \end{pmatrix}, \quad \boldsymbol{v}_3 = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \quad \boldsymbol{v}_4 = \begin{pmatrix} -1 \\ 3 \\ 1 \\ 0 \end{pmatrix}.$$

5. A matrix is called *singular* if the homogeneous linear system $A\boldsymbol{v} = \boldsymbol{b}$ has a "non-trivial" solution $\boldsymbol{v} \neq 0$.

   Prove that $AB = 0$ implies that at least one of the matrices is singular.