Calculus and Elements of Linear Algebra I

Homework 4

Due on Moodle, Monday, October 5, 2020

1. Consider the polynomial
$$p(x) = -2 + 9\,x - 6\,x^2 + x^3\,.$$
   (a) Find the roots of p.
   Hint: One of the roots is $x = 2$.
   (b) Compute $p'(x)$ and find its roots.
   (c) Compute $p''(x)$ and find its roots.
   (d) Do you see a pattern? Explain.

2. Compute the derivatives of the following functions.
   (a) $f(x) = \dfrac{x}{a + x^2}$ where a is a constant
   (b) $g(t) = \cos(\omega t + \phi)$ where ω and ϕ are constants
   (c) $h(s) = \sin(s^3)$
   (d) $j(s) = (\sin s)^3$
   (e) $k(x) = \ln(x^a + x^{-a})$ where $a \neq 0$ is a constant
   Note: You can use without further discussion that $(\ln x)' = 1/x$.
   (f) $\ell(x) = \ln(a^x + a^{-x})$ where $a > 0$ is a constant
   (g) $u(x) = \exp(bx)$ where b is a constant
   (h) $v(x) = x^2 \exp(x)$
   (i) $w(x) = \exp(-x^2)$
   (j) $z(x) = x^x$

3. Use the definition of the derivative,
$$f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}\,,$$
to show that the function $f(x) = |x|$ is not differentiable at $x = 0$.

4. Suppose that $f\colon (a,b) \to \mathbb{R}$ is differentiable at $x \in (a,b)$ with $f'(x) \neq 0$ and that f is invertible in some neighborhood of x.

   (a) Show that the inverse function f^{-1} is differentiable in some neighborhood of $y = f(x)$ with

   $$(f^{-1})'(y) = \frac{1}{f'(x)}\,.$$

   (b) Give a geometrical explanation of why this is true.