Calculus and Elements of Linear Algebra I

Homework 6

Due on Moodle, Monday, October 19, 2020

In Problems 1–3, your answer should include the following points:

(i) The domain of the function,

(ii) y-intercept if any, x-intercepts when they are easy to compute,

(iii) horizontal asymptotes,

(iv) vertical asymptotes,

(v) analysis of the first derivative (intervals where the function is increasing or decreasing, local minima or maxima)

(vi) analysis of the second derivative (intervals where the function is concave up or concave down, points of inflection).

Finally, sketch the function. Your drawing does not need to be to scale, but should show all the qualitative features of the graph.

1. Sketch the graph of the function
$$f(x) = \frac{x^2}{x^2 - 1}\,.$$

2. Sketch the graph of the function
$$f(x) = \sqrt{x} - \ln x\,.$$

3. Sketch the graph of the function
$$f(x) = e^{-1/x}\,.$$

4. A farmer owns an area of land adjacent to a lake with a straight shoreline. She has $500\,\mathrm{m}$ of fencing with which to enclose her cattle. Assuming that she uses the shore as one side of a rectangular enclosure, what dimensions should the rectangle have in order that the cattle have the largest possible area in which to graze?

5. Find the radius of the circle which minimizes the square of the distance of the circle centered at the origin to the points $(0, 1)$ and $(-1, 2)$.