Calculus and Elements of Linear Algebra I

Homework 7

Due on Moodle, Monday, October 26, 2020

1. Use substitution to evaluate the following integrals.

   (a) $\displaystyle\int \frac{\cos(\pi/x)}{x^2}\,\mathrm{d}x$

   (b) $\displaystyle\int_1^{\mathrm{e}} \frac{\ln x}{x}\,\mathrm{d}x$

2. Use integration by parts to evaluate the following integrals.

   (a) $\displaystyle\int \sin(x)\,\ln(\cos x)\,\mathrm{d}x$

   (b) $\displaystyle\int_0^{\pi/2} x\,\sin(x)\,\cos(x)\,\mathrm{d}x$

3. (a) Prove the reduction formula

$$\int \cos^n(x)\,\mathrm{d}x = \frac{1}{n}\,\cos^{n-1}(x)\,\sin(x) + \frac{n-1}{n}\int \cos^{n-2}(x)\,\mathrm{d}x\,.$$

   Hint: Use integration by parts and the fact that $\cos^2(x) + \sin^2(x) = 1$.

   (b) Evaluate $\displaystyle\int \cos^2(x)\,\mathrm{d}x$.

   (c) Evaluate $\displaystyle\int_0^{2\pi} \cos^4(x)\,\mathrm{d}x$.

4. Find the area between the curves $x = 1 - y^2$ and $y = -x - 1$.

5. Suppose that $f\colon \mathbb{R} \to \mathbb{R}$ continuous and *odd*, i.e., satisfies $-f(x) = f(-x)$. Show that

$$\int_{-a}^{a} f(x)\,\mathrm{d}x = 0\,.$$