

Calculus and Elements of Linear Algebra I

Mock Midterm Exam

Monday, November 2, 2020

1. Compute the following limits, if they exist. Else, argue why the limit does not exist.

(a) $\lim_{s \rightarrow -1} \frac{\frac{1}{s} - 1}{s^3 - 1}$

(b) $\lim_{x \rightarrow \infty} \frac{e^{2x} + x^3 + \ln x}{3e^{2x} - x^3 + \cos x}$

(c) $\lim_{r \rightarrow 1} \frac{|r - 1|}{r^2 - 1}$

(5+5+5)

2. The function $f(x)$ is defined on the interval $[0, 2]$ and is between $4 - x$ and $x^2 + 2$ for all x in this interval. Does it have to be continuous at $x = 1$? Explain why or why not. (5)

3. Show that the equation $x^7 - 3x - 1 = 0$ has at least one solution in the interval $[-1, 1]$. (5)

4. (a) Show that

$$\frac{d}{dx} \arctan x = \frac{1}{1 + x^2}.$$

- (b) Consider the function

$$f(x) = 2 \arctan x - x.$$

Find its domain, horizontal and vertical asymptotes, local minima, local maxima, and inflection points of f . Identify the regions where the graph of f is concave upward or concave downward. Finally, sketch the graph of the function.

(5+10)

5. An airplane is flying towards a radar station at a constant height of 6 km above the ground. The distance s between the airplane and the radar station is decreasing at a rate of 400 km/h when $s = 10$ km. What is the horizontal speed of the plane? (10)

6. Compute the following definite or indefinite integrals.

(a) $\int x^{-3} e^{1/x} dx$

(b) $\int \frac{x+1}{x^2(x^2+1)} dx$

(c) $\int_0^{2\pi} (\cos^2 \phi - \sin^2 \phi) d\phi$

(10+10+5)

7. Find the derivative of the function

$$F(x) = \int_{\sqrt{x}}^x \frac{e^t}{t} dt.$$

(5)