Homework 2 Solutions:

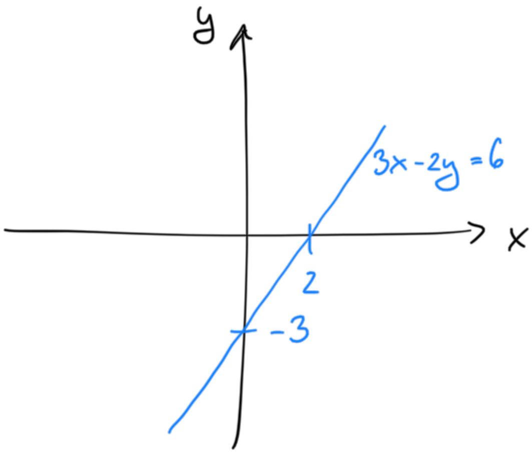
1(a). $3x - 2y = 6$

The solution set of a linear equation is a line.

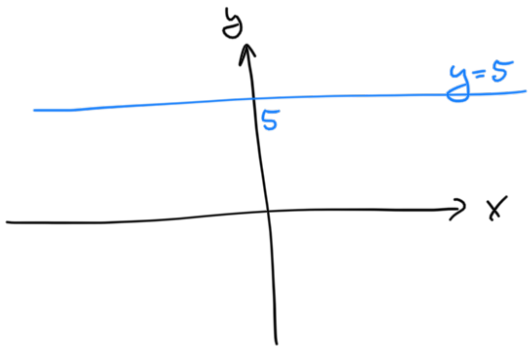
Here, it is easiest to compute the x- and y- intercepts:

x- intercept: set $y=0 \Rightarrow 3x = 6 \Rightarrow x = 2$

y- intercept: set $x=0 \Rightarrow -2y = 6 \Rightarrow y = -3$



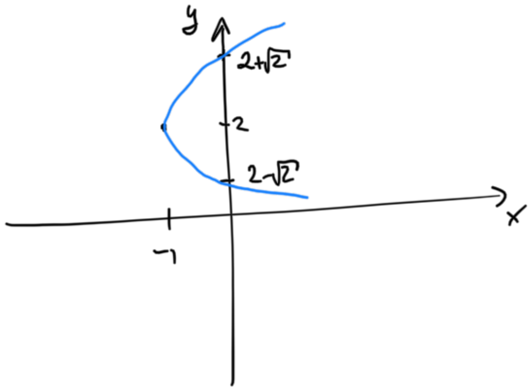
(b)



(c) $(y-2)^2 = 2(x+1)$

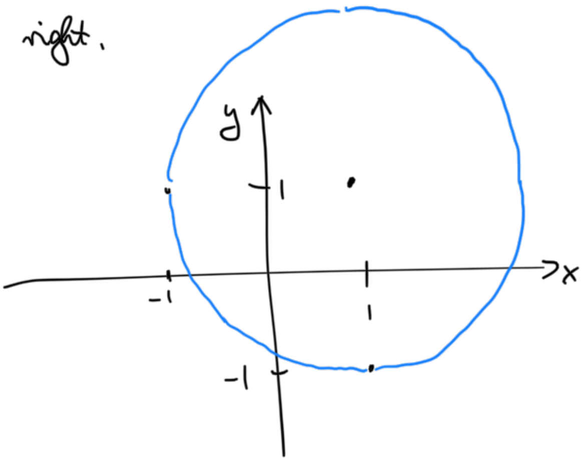
It's a parabola, open to the right, shifted one unit left and 2 units up, wider than the standard parabola.

Compute y- intercepts: $(y-2)^2 = 2 \Rightarrow y = 2 \pm \sqrt{2}$



(d) $(x-1)^2 + (y-1)^2 = 4$

It's a circle of radius $\sqrt{4} = 2$, shifted one unit up and one unit right.



<u>Note:</u> Precise drawing to scale, or computer-generated plots are not required (but of course accepted). It suffices to provide free-hand drawings as shown above that show the <u>qualitative</u> features of the graphs.

2(a): $3x - 2y = 6 \Rightarrow 2y = 3x - 6 \Rightarrow y = \underbrace{\frac{3}{2}x - 3}_{f(x)}$

(b) $y = 3$, so $f(x) = 3$

(c) Fails the vertical line test.

(However, if we restrict, e.g., to the upper branch of the parabola, we can write

$$y - 2 = \sqrt{2(x+1)} \Rightarrow y = 2 + \sqrt{2(x+1)}$$

which is a function with domain $[-1, \infty)$.) Not required

(d) Fails the vertical line test.

(However, if we restrict, e.g., to the lower semi-circle, we can write

$$y - 1 = -\sqrt{4 - (x-1)^2} \Rightarrow y = 1 - \sqrt{3 + 2x - x^2}$$

which is a function with domain $[-1, 3]$.) Not required.

3(a). $3x - 2y = 6 \Rightarrow 3x = 6 + 2y \Rightarrow x = \underbrace{2 + \frac{2}{3}y}_{g(y)}$

(b) Fails the horizontal line test

(c) $x = \underbrace{\frac{1}{2}(y-2)^2 - 1}_{g(y)}$

(d) Fails the horizontal line test.

(But can be made a function by restricting to a left or right semi-circle as in 2d.)

4. $\underbrace{y = 5^{x-2}}_{=f(x)} \Rightarrow \log_5 y = x-2 \Rightarrow x = \underbrace{2 + \log_5 y}_{=f^{-1}(y)}$

$\text{Domain}(f) = \mathbb{R} = \text{Range}(f^{-1})$

$\text{Domain}(f^{-1}) = (0,\infty) = \text{Range}(f)$

5(a). $\lim_{x\to 4} x^2+5x-5 = 4^2 + 5\cdot 4 - 5 = 16+15 = 31$

(b) $\lim_{s\to 3} \frac{s^2-9}{s-3} = \lim_{s\to 3} \frac{(s-3)(s+3)}{s-3} = \lim_{s\to 3} s+3 = 6$

(c) $\lim_{t\to 0} \frac{t^2}{t} = \lim_{t\to 0} t = 0$

(d) $\lim_{w\to 3} \frac{\frac{1}{w}-\frac{1}{3}}{w-3} = \lim_{w\to 3} \frac{\frac{3-w}{3w}}{w-3} = \lim_{w\to 3} \frac{-1}{3w} = -\frac{1}{9}$

(e) $\lim_{h\to 0} \frac{\sqrt{1+h}-\sqrt{1-h}}{h} = \lim_{h\to 0} \frac{\sqrt{1+h}-\sqrt{1-h}}{h} \cdot \frac{\sqrt{1+h}+\sqrt{1-h}}{\sqrt{1+h}+\sqrt{1-h}}$

$= \lim_{h\to 0} \frac{\cancel{1}+h-(\cancel{1}-h)}{h(\sqrt{1+h}+\sqrt{1-h})} = 2\lim_{h\to 0} \frac{1}{\sqrt{1+h}+\sqrt{1-h}} = \frac{2}{2} = 1$