

Functional Analysis

Homework 3

due March 4, 2009

1. Let X be a normed vector space and $C \subset X$ an open, convex subset containing the origin. Define $p: X \rightarrow [0, \infty)$ via

$$p(x) = \inf\{\alpha > 0: x/\alpha \in C\}.$$

Prove that

- (a) p is sublinear;
- (b) p is bounded, i.e. there exists $m > 0$ such that for every $x \in X$,

$$0 \leq p(x) \leq m \|x\|;$$

- (c) $C = \{x \in X: p(x) < 1\}$.

2. Folland, p. 164–165, no. 27, 34, 35, 37, 38.