General Mathematics and ACM II

Exercise 5

February 18, 2011

1. (Ivanov, p. 34, Problem 6.) A billiard ball sits against the cushion of a billiard table. In which direction should the ball be cued so that after bouncing off the three other sides of the table, it returns to its point of departure?

2. (Ivanov, p. 35, Exercise.) Prove that the composite of two axial symmetries (an *axial symmetry* is a reflection about a line, the *axis*) with intersecting axes of symmetry is a rotation about the point of intersection of these axes through an angle equal to twice the angle between them.

3. A *Euclidean transformation* F or, in short, a *motion* of the plane is a map which preserves the Euclidean distance between points, i.e.

$$|F(a) - F(b)| = |a - b|$$

   where $|a - b|$ denotes the distance between two arbitrary points a and b.

   Show that every motion of the plane is bijective.

   Hint: To prove that it is surjective, notice that the transformation must map lines onto lines and circles onto circles.