

General Mathematics and ACM II

Exercise 10

March 18, 2011

1. Newton's second law of mechanics for a particle of mass m situated at position $x(t)$ moving with velocity $v(t)$ and subject to a force $F(x(t))$ can be written

$$\begin{aligned}\frac{dx}{dt} &= v, \\ m \frac{dv}{dt} &= F(x(t)).\end{aligned}$$

Use the chain rule of calculus to show that the particle satisfies the same equation with t replaced by the reversed time $r = -t$ and v replaced by $-v$.

2. Show that in a time-discrete, reversible, system with a finite number of states any orbit must return to its initial state after a finite number of steps.
3. In the Kac ring model, N sites are placed around a circle. The sites are populated with B black balls and $W = N - B$ white balls at random. Moreover, n markers are placed on the edges between the sites at random; the number of black balls just before a marked edge is denoted b , the number of white balls just before a marked edge is w . Let μ denote the probability that an edge has a marker on it. Explain why

$$\mu = \frac{n}{N} = \frac{b}{B} = \frac{w}{W}.$$