General Mathematics and ACM II

Exercise 16

April 8, 2011

This exercise sheet is due Friday, April 29, 2011.

1. Find all solutions for the underdetermined linear system $A\boldsymbol{x} = \boldsymbol{b}$, where

$$A = \begin{pmatrix} 2 & 2 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & -1 \\ 3 & 3 & 2 & 1 \end{pmatrix} \qquad \text{and} \qquad \boldsymbol{b} = \begin{pmatrix} -1 \\ 1 \\ -2 \\ 0 \end{pmatrix}.$$

2. Consider a linear programming problem with n variables in standard form. Explain why a non-negative solution to $m \leq n$ equality constraints in which at least $n - m$ variables are zero represents a vertex of the feasible region.

   Hint: Recall how the general solution to an underdetermined system of linear equations looks like, cf. Question 1.

3. Write the following linear programming problem in its standard form.

   Maximize

$$z = 2\,x_1 - x_2 + x_3$$

   subject to

$$\begin{aligned} x_1 - x_2 &\leq 1\,, \\ x_2 - x_3 &\geq 1\,, \\ x_3 - x_1 &\leq 3\,, \\ x_1 &\geq -2\,, \\ x_2 &\leq 1\,, \\ x_3 &\leq 0\,. \end{aligned}$$

4. Solve the linear programming problem from Question 3 using the simplex method.

5. Solve the following linear programming problem using the simplex method.

   Maximize

$$z = 3\,x_1 + 4\,x_2$$

   subject to

$$2\,x_1 + x_2 \leq 4\,,$$
$$3\,x_1 + 2\,x_2 \leq 8\,,$$
$$x_i \geq 0 \text{ for } i = 1, 2\,.$$