General Mathematics and ACM II

Exercise 17

April 29, 2011

1. Show that the leaving variable in one iteration of the simplex method can never be the entering variable in the next iteration.

2. The *primal* form of a linear programming problem is

$$\begin{aligned} &\text{minimize } \boldsymbol{c}^T \boldsymbol{x} \\ &\text{subject to } A\boldsymbol{x} = \boldsymbol{b}, \boldsymbol{x} \geq 0 \,. \end{aligned} \tag{P}$$

   The corresponding dual problem reads

$$\begin{aligned} &\text{maximize } \boldsymbol{y}^T \boldsymbol{b} \\ &\text{subject to } \boldsymbol{y}^T A \leq \boldsymbol{c}^T \,. \end{aligned} \tag{D}$$

   Here, A is an $m \times n$ matrix, $\boldsymbol{x}, \boldsymbol{c} \in \mathbb{R}^n$, and $\boldsymbol{y}, \boldsymbol{b} \in \mathbb{R}^m$.

   Show that if $\boldsymbol{x}$ solves (P) and $\boldsymbol{y}$ solves (D), then

$$\boldsymbol{y}^T \boldsymbol{b} \leq \boldsymbol{c}^T \boldsymbol{x} \,.$$

   Conclude that the primal problem does not have a finite minimum if and only if the feasible region of the dual problem is empty.

3. In the notation of the previous question, show that if $\boldsymbol{x}$ is feasible for problem (P) and $\boldsymbol{y}$ is feasible for problem (D), and if furthermore

$$\boldsymbol{y}^T \boldsymbol{b} = \boldsymbol{c}^T \boldsymbol{x} \,,$$

   then $\boldsymbol{x}$ solves (P) and $\boldsymbol{y}$ solves (D).