General Mathematics and CPS II

Exercise 9

March 2, 2012

1. Let G be a group, and let H and K be subgroups of G. Show that $H \cap K$ is also a subgroup of G.

2. Prove Ivanov p. 40, Lemma 4. I.e., show that the symmetry group of an ornament may contain, apart from the cyclic subgroup of translations $\langle \Pi \rangle$, only the following:

   (a) A line reflection $R_{\|}$ about the axis of the ornament.

   (b) Line reflections about axes perpendicular to the ornament.

   (c) Point reflections about points on the axis of the ornament.

   (d) At most one *glide reflection* about the axis of the ornament, which is a composition of a translation and a line reflection about the axis of the ornament.

3. (Ivanov, p. 41, Exercise.) Let R_α denote the reflection about the line $x = \alpha$. Let G be the (symmetry) group generated by the unit translation along the x-axis and by R_0. Show that $R_\alpha \in G$ if and only if $2\alpha \in \mathbb{Z}$.