Engineering and Science Mathematics 2B

Homework 10 - Quiz Only!

Quiz dates are Tuesday, May 14 or Thursday, May 16

Normal questions and advanced questions (A) are worth 5 points; easy questions (E) are worth 4 points. Complete either the easy, or the advanced version, not both.

1. Consider an infinite sequence of independent trials which result in *success* with probability p and in *failure* with probability $q = p - 1$.

   Let X be the number of trials required to obtain the first success. Find $P(X = n)$, i.e., the probability that n trials are required to obtain the first success.

2. Consider an infinite sequence of independent trials which result in *success* with probability p and in *failure* with probability $q = p - 1$.

   Let X be the number of failures before the rth success. Find $P(X = n)$, i.e., the probability that there are n failures before the rth success.

3. Show that the moment generating function for the binomial distribution is given by

   $$M(t) = E[\mathrm{e}^{tX}] = (p\mathrm{e}^t + q)^n$$

   and use the moment generating function to find $E[X]$.

4. In a promotional quiz each player, independently, has the chance of winning 1 000 Euros with probability p. It is obviously bad for the organizer if more than one person wins, but it is also considered bad for the promotion if nobody wins at all.

   (a) With 500 players participating, what is the optimal choice for p, i.e. the one that maximizes the chance of exactly one player winning?

   (b) For this value of p, what is the expected payout?

5. Suppose you receive, on average, 5 emails per hour. What is the probability of receiving only one mail in one hour?

6. Let $X_1, X_2, \ldots$ be independent discrete random variables taking values in $\{-\frac{1}{2}, \frac{1}{2}\}$ with probability function

   $$p(-\tfrac{1}{2}) = q\,, \qquad p(\tfrac{1}{2}) = (1 - q)\,,$$

   and $p(x) = 0$ for all other values of x.

(E) Find the expectation for the random variable

$$Z_N = \frac{X_1 + \cdots + X_N}{N}$$

as $N \to \infty$.

(A) Let $Y_n = 2^{-n} X_n$. Find the expectation for the random variable

$$Z_N = Y_1 + \cdots + Y_N$$

as $N \to \infty$.