Engineering and Science Mathematics 2B

Homework 2

due February 20, 2013, before 12:00

Normal questions and advanced questions (A) are worth 5 points; easy questions (E) are worth 4 points. Complete either the easy, or the advanced version, not both.

1. A matrix A is called normal if $AA^H = A^H A$, where A^H denotes the Hermitian conjugate of A. Show (a) that $(A^H)^{-1} = (A^{-1})^H$, and (b) that A^{-1} is normal if A is normal.

2. Let
$$B = \begin{pmatrix} 0 & -\mathrm{i} & \mathrm{i} \\ \mathrm{i} & 0 & -\mathrm{i} \\ -\mathrm{i} & \mathrm{i} & 0 \end{pmatrix}, \qquad C = \frac{1}{\sqrt{8}} \begin{pmatrix} \sqrt{3} & -\sqrt{2} & -\sqrt{3} \\ 1 & \sqrt{6} & -1 \\ 2 & 0 & 2 \end{pmatrix}.$$
Are these matrices real, diagonal, symmetric, antisymmetric, singular, orthogonal, Hermitian, anti-Hermitian, unitary, and/or normal? (Recall that a matrix is normal if $AA^H = A^H A$.)

3. (E) Let
$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 0 \\ 3 & 4 \end{pmatrix}.$$
Compute AB and BA. Conclude that (i) $AB \neq BA$, and (ii) that $AB = 0$ does not imply that either A or B is the zero matrix.

   (A) Prove that $AB = 0$ implies that at least one of the matrices is singular.

4. (E) Determine if the following set of vectors is linearly dependent or linearly independent:
$$\begin{pmatrix} -4 \\ 0 \\ 1 \\ 5 \end{pmatrix}, \quad \begin{pmatrix} -3 \\ -1 \\ 0 \\ 4 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 4 \\ 3 \\ 6 \end{pmatrix}.$$

   (A) Prove the following statement: Let $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n$ be linearly independent. If a vector $\boldsymbol{w}$ can be written
$$\boldsymbol{w} = \sum_{k=1}^{n} \alpha_k \boldsymbol{v}_k,$$
the choice of the coefficients $\alpha_1, \ldots, \alpha_n$ is unique.

5. Solve the following system of linear equations using the method taught in class.

$$\begin{aligned} x_1 + 3\,x_2 - 5\,x_3 &= 4 \\ x_1 + 4\,x_2 - 8\,x_3 &= 7 \\ -3\,x_1 - 7\,x_2 + 9\,x_3 &= -6 \end{aligned}$$

6. (E) Solve the following system of linear equations using the method taught in class.

$$\begin{aligned} x_1 + 3\,x_2 + x_3 &= 1 \\ -4\,x_1 - 9\,x_2 + 2\,x_3 &= -1 \\ -3\,x_2 - 6\,x_3 &= -3 \end{aligned}$$

(A) Find conditions on α such that following system of linear equations has (a) exactly one solution, (b) no solutions, or (c) an infinite number of solutions; give all solutions where they exist.

$$\begin{aligned} x_1 + \alpha\,x_2 &= 1 \\ x_1 - x_2 + 3\,x_3 &= -1 \\ 2\,x_1 - 2\,x_2 + \alpha\,x_3 &= -2 \end{aligned}$$