Introduction to Partial Differential Equations

Homework 2

due February 21, 2013

1. Show that
$$\Delta u = 2n \lim_{\varepsilon \to 0} \frac{1}{\varepsilon^2} \fint_{\partial B(x,\varepsilon)} (u(y) - u(x))\, dS(y)\,.$$

2. Evans, p. 85 problem 3.

   Note: This problem can be hard when you try it. Please ask in class in case you get stuck.

3. (a) The *standard mollifier* is defined by
$$\eta(x) \equiv \begin{cases} c(n)\, \exp\left(\dfrac{1}{|x|^2 - 1}\right) & \text{if } |x| < 1 \\ 0 & \text{otherwise}\,, \end{cases}$$
   where $c(n)$ is chosen such that
$$\int_{\mathbb{R}^n} \eta(x)\, dx = 1\,.$$
   Show that $\eta \in C^\infty(\mathbb{R}^n)$.

   (b) Show that if η_ε is a radial mollifier, and u is a radial, locally integrable function, then its mollification
$$u_\varepsilon(x) = (\eta_\varepsilon * u)(x) = \int_{\mathbb{R}^n} \eta_\varepsilon(y)\, u(x - y)\, dy$$
   is also radial.