

# General Mathematics and CPS II

## Exercise 12

March 21, 2014

1. Kac ring paper, Exercise 5.

2. Show that

$$\sum_{k=0}^n \binom{n}{k} k x^k y^{n-k} = n x (x + y)^{n-1}$$

and

$$\sum_{k=0}^n \binom{n}{k} k^2 x^k y^{n-k} = n x (x + y)^{n-1} + n(n-1) x^2 (x + y)^{n-2}.$$

*Hint:* Differentiate the binomial theorem.

3. Consider an ensemble of Kac rings with  $N$  sites where each site is colored black independently with probability  $\nu$ . Show that

$$\langle \Delta \rangle = (2\nu - 1) N$$

and

$$\text{Var}[\Delta] = 4\nu(1 - \nu) N.$$

*Hint:* Use the result of Question 2.

*Remark:* This is in fact a direct consequence of a well known result on the variance of the binomial distribution which has a rather elegant proof in terms of *moment generating functions*. These are usually introduced in a first course on probability theory. You will likely encounter them this semester in Engineering and Science Mathematics 2A/B.