General Mathematics and Computational Science II

Midterm Exam

March 14, 2014

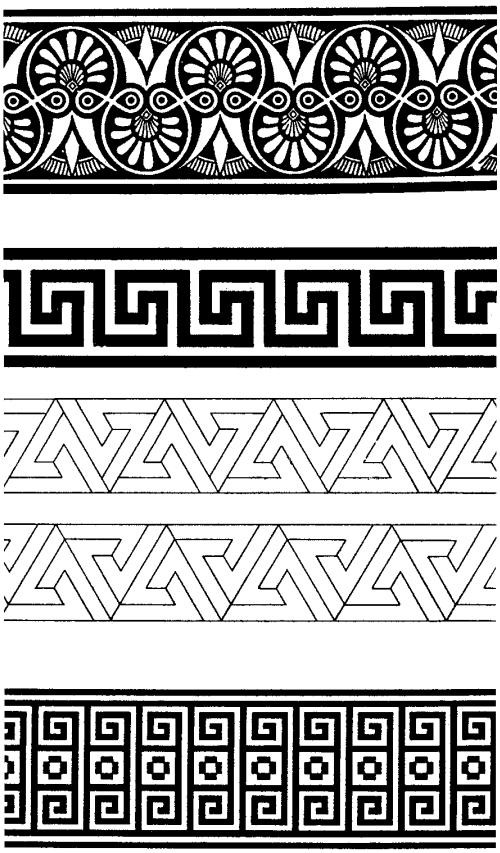
1. Show that if v is a vertex of odd valency in a finite graph, then there exists a path connecting v to another vertex u of odd valency. (10)

2. Show that a tree with more than one vertex is bipartite. (10)

3. The structure of chemical molecules can be considered as a graph. Show that the graph of pentanol, an alcohol with the chemical formula $C_5H_{11}OH$, is a tree.

   Note: Chemists know that Carbon (C), Oxygen (O), and Hydrogen (H) atoms have valencies 4, 2, and 1, respectively. Subscripts in the chemical formula denote the number of each atom in the molecule. (10)

4. Identify the symmetries and corresponding frieze groups for the following five ornaments. Further, clearly draw all centers and lines of symmetry into the picture.



(From R.N. Umble, *Transformational Plane Geometry*,
`http://www.millersville.edu/~rumble/Math.355/Book/TPG-Spring2012.pdf`.) (10)

5. (a) Show that $V = \{1, 3, 5, 7\}$ endowed with multiplication modulo 8 is a group.

   (b) Show that V is isomorphic to the dihedral group

$$D_2 = \langle a, b \mid a^2 = b^2 = e, ba = a^{-1}b \rangle .$$

   (c) Can you think of a realization of this group as the symmetry group of some geometric shape?

(4+3+3)

6. Let G be the group generated by Φ_A and Φ_B, two rotations about different centers of rotation A and B. Show that G contains a translation. (10)