

General Mathematics and CPS II

Exercise 8

February 27, 2015

1. A *Euclidean transformation* F or, in short, a *motion* of the plane is a map which preserves the Euclidean distance between points, i.e.

$$|F(a) - F(b)| = |a - b|$$

where $|a - b|$ denotes the distance between two arbitrary points a and b .

Show that every motion of the plane is bijective.

Hint: To prove that it is surjective, notice that the transformation must map circles onto circles.

2. (Ivanov, p. 39.) Recall that the symmetry group of a subset A of the plane is defined as

$$\text{Sym}(A) = \{F \text{ motion} : F(A) = A\}.$$

Prove that such a set of motions is indeed a group.

3. Let G be a group and let $a, b \in G$. Show that $(ab)^{-1} = b^{-1} a^{-1}$.