110231 Nonlinear Dynamics Lab

Exercise Sheet

'Excitable dynamics on graphs'

Let us consider a discrete state space Σ consisting of 'excited' E, 'susceptible' S and 'refractory' R. A susceptible element goes into the excited state, when it has an excited neighbor, $S \xrightarrow{E \in \text{NB}} E$, where NB stands for the neighborhood of an element; an excited element always enters the refractory state, $E \xrightarrow{1} R$; a refractory element has a probability p to enter the susceptible state, $R \xrightarrow{p} S$. Additionally, with a probability f an element can go from the susceptible state to the excited state, $S \xrightarrow{f} E$ (see also the previous exercise sheet 'Simple model of a neuron', where the same model was discussed in the deterministic limit p=1, f=0).
This minimal model of excitable dynamics can be seen as a symbolic encoding of the excitable behavior we observed in the FitzHugh-Nagumo oscillator (see Figure 1). Formally, this model is a stochastic cellular automaton.

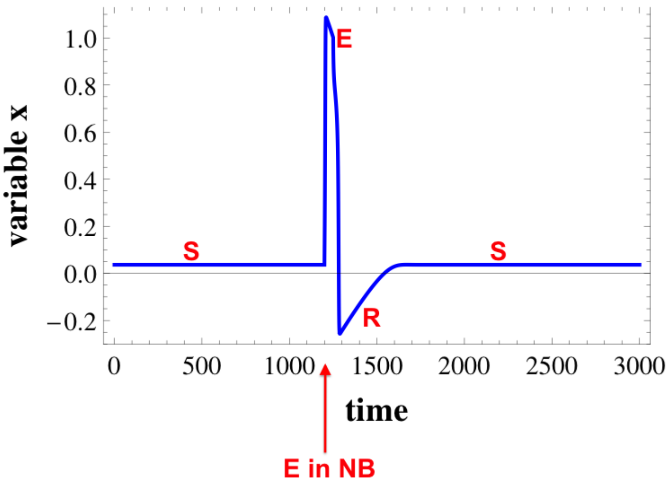

Figure 1: Excitable behavior of a FitzHugh-Nagumo oscillator. At time t=1000, a perturbation (which could correspond to an excitation of a neighboring element) moves the oscillator slightly ($\Delta x = 0.1$) away from its stable steady state. The oscillator responds with a spike, overshoots and returns to the fixed points. These phases correspond to the discrete states of the cellular-automaton model.

Exercise 1: Implement this minimal model and run it on an ER random graph (named after the mathematicians Paul Erdős and Alfréd Rényi). Qualitatively observe the changes of the dynamics as a function of connectivity (i.e., the number of links) and of the rate of spontaneous excitations, f. A suitable size for the ER graph is 100 nodes. The recovery rate p may be set to 0.2, while f should be varied logarithmically between 0.001 and 1. In *Mathematica*, ER random graphs can be generated with the `RandomGraph` command.

Exercise 2: Repeat the same analysis for a BA random graph (named after the statistical physicists Réka Albert and Albert-László Barabási). A BA graph can also be generated with the `RandomGraph` command, using the option `BarabasiAlbertGraphDistribution[n, k]`. Compare (e.g., by computing the correlation coefficient) the co-activation matrix (where an entry in the matrix is the percentage of *simultaneous* activations of two nodes) with the graph's adjacency matrix for different values of p and f.

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Exercise 3: Visualize the dynamics by plotting the graph and representing the states of all nodes at each time point as colors on the respective node; convert the sequence of these single-timepoint snapshots into a movie.

Additional task:

Exercise 4: Explore these excitable dynamics for $f = 0$ on a ring graph of n nodes, starting with n=3. Compare the deterministic case $p = 1$ with the stochastic case $p < 1$. Compute the success rate (long-time persistent activity) as a function of recovery probability p and ring length n.