Operations Research

Final Exam

May 21, 2016

1. Use the graphical method to minimize

$$z = x + y$$

subject to

$$x^2 + y^2 \geq 1\,,$$
$$x, y \geq 0\,.$$

(20)

2. Paula is using the Pyomo notebook displayed on the next page to optimize the shipment of containers, described by the decision variables x_{ij}, for her company.

   (a) Which problem is she solving? Describe the problem in words. You may complete the model scenario in any way consistent with the given information.

   (b) Which problem is she solving? State the problem in traditional mathematical notation.

   (c) A container must be shipped as a unit. Paula remembers the "integrality property" from her OR class and is surprised by the solution she is obtaining. Can you explain what is going on? How can she obtain an integer solution?

   (d) Give a *brief* interpretation of the program output shown in cells `[4]` and `[5]`.

(5+5+5+5)

3. Use dynamic programming to solve the following *integer* nonlinear programming problem.

$$\text{Minimize } z = 3\,x_1 + 2\,x_2^2 + x_3^3$$
$$\text{subject to } x_1 + x_2 + x_3 = 4\,,$$
$$x_1, x_2, x_3 \geq 0\,.$$

(20)

In [1]:
```
from pyomo.environ import *
from pyomo.opt import *
opt = solvers.SolverFactory("ipopt")
```

In [2]:
```
P = ['P1', 'P2', 'P3']
M = ['M1', 'M2', 'M3']

s = {'P1':3, 'P2':5, 'P3':1}
d = {'M1':2, 'M2':2, 'M3':5}

c = {('P1','M1'):10, ('P1','M2'):5,  ('P1','M3'):15,
     ('P2','M1'):15, ('P2','M2'):10, ('P2','M3'):20,
     ('P3','M1'):5,  ('P3','M2'):5,  ('P3','M3'):15}

model = ConcreteModel()
model.x = Var(P, M, within=NonNegativeReals)

model.z = Objective(expr = sum(c[i,j]*model.x[i,j] for i in P for j in
M), sense=minimize)

def supply_rule (model, i):
    return sum(model.x[i,j] for j in M) <= s[i]
model.supply = Constraint(P, rule=supply_rule)

def demand_rule (model, j):
    return sum(model.x[i,j] for i in P) >= d[j]
model.demand = Constraint(M, rule=demand_rule)

model.dual = Suffix(direction=Suffix.IMPORT)
results = opt.solve(model)
model.x.get_values()
```

Out[2]:
```
{('P1', 'M1'): 0.4634679251867636,
 ('P1', 'M2'): 0.8561172866483512,
 ('P1', 'M3'): 1.6804148178139067,
 ('P2', 'M1'): 0.5365320260507703,
 ('P2', 'M2'): 1.1438827029955108,
 ('P2', 'M3'): 3.3195851417491173,
 ('P3', 'M1'): 1.0000000288526543,
 ('P3', 'M2'): 0.0,
 ('P3', 'M3'): 0.0}
```

In [3]:
```
model.z.expr()
```

Out[3]:
```
124.99999835228026
```

In [4]:
```
for j in M:
    print(j, model.dual[model.demand[j]])
```

```
M1 15.043624766149037
M2 10.043624768629797
M3 20.043624770065662
```

In [5]:
```
for i in P:
    print(i, model.dual[model.supply[i]])
```

```
P1 -5.043624771556779
P2 -0.043624770820542835
P3 -10.043624768654922
```

4. Customs authorities have busted a smuggling ring specialized in exclusive watches. Authorities are now selling off the seized merchandise for EUR 250 a piece. It is known that the true value of each hand-made watch is EUR 2000, but also that there are many cheap replicas in circulation with a market value of only EUR 40 a piece. The conditions of sale do not provide any warranty whether a watch is genuinely hand-made. Prior to purchase, only a visual inspection is allowed. A lay person will not be able to tell a genuinely hand-made watch from a replica.

   (a) The trade press has recently reported that 90% of all watches circulating on the black market are replicas. Given this information, would you choose to acquire watches at the quoted price?

   (b) An expert is offering her services to do a visual pre-screening of the watches. In the past, she has been correct 75% of the time. Compute the probability that she will classify one of the watches as genuinely hand-made.

   (c) Given that the expert classifies a watch as hand-made, what is the probability that the watch is truly hand-made?

   (d) What is the maximal fee per watch you should be willing to pay for her expert evaluation?

(5+5+5+5)

5. An airplane manufacturer is contracted to produce a small number of a particular type of airplane during the coming years. The manufacturer will need to decide each year whether to set up a production run with a fixed set-up cost of EUR 1 000 000 per run. During each production run, the manufacturer can make at most 6 airplanes. If an airplane is not delivered during the year it is produced, it will incur a holding cost of EUR 100 000 per year. The number of airplanes required are $r_1 = 1$, $r_2 = 6$, $r_3 = 2$, and $r_4 = 3$ during each of the years.

   Which production schedule(s) minimize the total cost for setup and storage? (20)

6. Consider an inventory under the assumptions of the economic order quantity (EOQ) model with replacement at a constant rate, namely:

   - Constant demand rate of d units per time,
   - continuous review with no planned shortages,
   - an order with batch size Q is used to replenish the inventory at a constant rate $r > d$ units per time where replenishments and withdrawals can happen simultaneously,
   - each order incurs a setup cost K per batch,
   - items in the inventory incur a holding cost h per unit per time.

   Derive the economic order quantity Q^* as a function of r, d, h, and K.

(20)