Introduction to Partial Differential Equations

Homework 1

due February 20, 2017

1. Evans, p. 85 problem 2

2. Consider a function of one complex variable $w\colon \mathbb{C} \to \mathbb{C}$ on an open connected subset of the complex plane, and write $w = w(z)$ with $w = u + \mathrm{i}v$ and $z = x + \mathrm{i}y$. The function w is called *(complex) differentiable* or *holomorphic* if u and v have continuous first partial derivatives with respect to x and y that satisfy the so-called *Cauchy–Riemann equations*

$$\begin{aligned} u_x &= v_y \\ u_y &= -v_x \end{aligned}$$

   It is known that holomorphic functions are infinitely differentiable.

   Show that the real and imaginary parts of a holomorphic function are harmonic.

3. Find all radial solutions for the modified Laplace equation

$$D \cdot (r\, Du(x)) = 0\,,$$

   where $u\colon \mathbb{R}^n \to \mathbb{R}$ with $n \geq 2$, and $r = |x|$.

4. Show that

$$\Delta u = 2n \lim_{\varepsilon \to 0} \frac{1}{\varepsilon^2} \fint_{\partial B(x,\varepsilon)} (u(y) - u(x))\, dS(y)\,.$$

5. Evans, p. 85 problem 3.

   Note: This problem can be hard when you try it. Please ask in class in case you get stuck.