Introduction to Partial Differential Equations

Homework 5

due May 15, 2017

1. Evans, p. 290 problem 5.
2. Evans, p. 290 problem 8.
3. Evans, p. 290 problem 9.
4. Show that, for every $u \in L^r(\mathbb{T})$ with $2 \leq r < \infty$,

$$\|u\|_{L^2} \leq (2\pi)^{\frac{r-2}{2r}} \|u\|_{L^r} .$$

   Hint: Hölder inequality.

5. Consider the Fisher–Kolmogorov equation on $\mathbb{T}$,

$$\begin{aligned} u_t &= u_{xx} + (1-u)\, u^m \,, \\ u(0) &= u^{\text{in}} \,, \end{aligned}$$

   where m is an even positive integer. Use the result from the previous question to sharpen the L^2 estimate derived in the lecture as follows: Show that

$$\limsup_{t \to \infty} \|u(t)\|_{L^2} \leq C$$

   where an explicit estimate for C can be given which, in particular, shows that C does not depend on the initial data u^{in}.

6. In the setting of the Question 5, use the result from Question 2 to prove that

$$\limsup_{t \to \infty} \|u(t)\|_{H^1} \leq C$$

   where an explicit estimate for C can be given which, in particular, shows that C does not depend on the initial data u^{in}. You may assume that u is sufficiently differentiable so that all your formal manipulations are justified.