Applied Differential Equations and Modeling

Homework 1

Due in class Tuesday, February 13, 2018

1. For each of the following,
   (a) Draw a direction field for the given differential equation
   (b) Based on the inspection of the direction field, describe how solutions behave for large t.
   (c) Find the general solution of the given differential equation, and use it to determine how solutions behave as $t \to \infty$.

$$2y' + y = 3t \tag{1}$$
$$ty' - y = t^2 \mathrm{e}^{-1} \tag{2}$$

2. Find the solution of the given initial value problem.
   (a) $y' - y = 2t\mathrm{e}^{2t}, \quad y(0) = 1$
   (b) $ty' + 2y = t^2 - t + 1, \quad y(1) = \frac{1}{2}, \quad t > 0$
   (c) $ty' + 2y = \sin t, \quad y(\pi/2) = 1, \quad t > 0$
   (d) $y' = (\mathrm{e}^{-x} - \mathrm{e}^{x})/(3 + 4y), \quad y(0) = 1$

3. In each of the following problems, find the critical value for the initial value a where the solution changes from one type of behavior to another. Describe both types of behavior in words.
   (a) $y' - \frac{1}{2}y = 2\cos t, \quad y(0) = a$
   (b) $2y' - y = \mathrm{e}^{t/3}, \quad y(0) = a$

4. Solve the initial value problem

$$y' = 2y^2 + xy^2, \quad y(0) = 1$$

and determine where the solution attains its minimum value.