

# Advanced Calculus and Methods of Mathematical Physics

Review for Final Exam

Exam Date: May 25, 9:00–11:00

1. Point-set topology: know the definitions of open, closed, and compact sets, and their behavior under continuous maps.

See, e.g., Problem 1 from the Analysis II Review Sheet, Spring Semester 2019.

2. Differentiation on normed vector spaces.

See, e.g., Question 5 on the Analysis II Final Exam, Spring Semester 2019; Question 4 on Mock Midterm Exam.

3. Inverse and implicit function theorem.

Homework 5, in particular Questions 1, 4; Mock Midterm Exam, Question 5; Problem 1 from the Analysis II Review Sheet, Spring Semester 2019.

4. Lagrange multipliers.

E.g., Problem 7 from the Analysis II Review Sheet, Spring Semester 2019.

5. Power series, radius of convergence, Laurent series.

E.g., Homework 10 Problem 5, Problem 4 from the Analysis II Review Sheet, Spring Semester 2019.

6. Change of variable formula.

E.g., use the change of variables formula to compute the area of the image of the unit square under the map

$$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x + y \\ y^2 - x \end{pmatrix}.$$

7. Line and surface integrals; Green's theorem, the divergence theorem, and Stokes' theorem; conservative vector fields. **This topic area will be a major part of the exam.**

All of Homework 8, 9 and similar questions are highly relevant.

8. Fourier series: elementary properties.

Elementary questions only, e.g. Homework 10 Questions 1 and 2.

9. Complex variables. Cauchy Riemann equations, setting up a complex line integral, Cauchy theorem, residue theorem.

Elementary questions only, e.g. Homework 10 Questions 3 and 5.