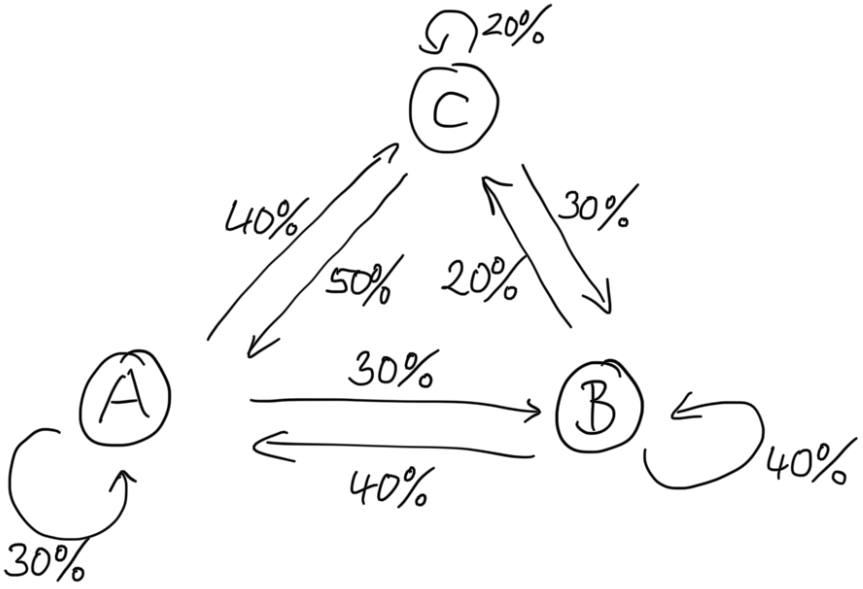
Review sheet, item 6, Example problem



Transfer Matrix:

$$P = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} \frac{3}{10} & \frac{3}{10} & \frac{4}{10} \\ \frac{4}{10} & \frac{4}{10} & \frac{2}{10} \\ \frac{5}{10} & \frac{3}{10} & \frac{2}{10} \end{pmatrix} \end{array}$$

(from \ to)

All entries are strictly positive $\Rightarrow$ chain is regular

Stationary state:

$$x^T P = X^T \quad \text{or} \quad P^T x = x \quad \Rightarrow \quad (P^T - I)x = 0$$

$$P^T - I = \begin{pmatrix} -\frac{7}{10} & \frac{4}{10} & \frac{5}{10} \\ \frac{3}{10} & -\frac{6}{10} & \frac{3}{10} \\ \frac{4}{10} & \frac{2}{10} & -\frac{8}{10} \end{pmatrix}$$

Divide out common factors from each row and continue Gauss elimination:

$$\begin{pmatrix} 1 & -7 & 4 & 5 \end{pmatrix} \xrightarrow{R2 \to R1} \begin{pmatrix} 1 & -2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & -4 \end{pmatrix} \xrightarrow[-2R2+R3\to R3]{7R2+R1\to R2} \begin{pmatrix} 0 & -10 & 12 \\ 0 & 5 & -6 \end{pmatrix}$$

$$\xrightarrow[\frac{2}{5}R3+R1\to R1]{\frac{1}{2}R2+R3\to R3} \begin{pmatrix} 1 & 0 & -\frac{7}{5} \\ 0 & 1 & -\frac{6}{5} \\ 0 & 0 & 0 \end{pmatrix}$$

$\Rightarrow$ Homogeneous solution is proportional to $\begin{pmatrix} -\frac{7}{5} \\ -\frac{6}{5} \\ -1 \end{pmatrix}$ or $\begin{pmatrix} 7 \\ 6 \\ 5 \end{pmatrix}$

Normalizing to get a probability vector, we find

$$X = \frac{1}{7+6+5}\begin{pmatrix} 7 \\ 6 \\ 5 \end{pmatrix} = \begin{pmatrix} \frac{7}{18} \\ \frac{6}{18} \\ \frac{5}{18} \end{pmatrix} \approx \begin{pmatrix} 0.39 \\ 0.33 \\ 0.28 \end{pmatrix}$$

$\Rightarrow$ The driver spends 39% of nights in Amsterdam,
33% " " " Berlin,
28% " " " Copenhagen.