Finite Mathematics

Homework 2

Due in class Wednesday, February 19, 2020

Note: Assignments marked (*) are *not* for bonus credit. They will not be graded. Do not turn them in. However, they will be discussed in the tutorial and typically example solutions are available online (in case of Hefferon's book, there is a PDF with solutions available on the book's web site.)

1. *Given the matrices

$$\boldsymbol{A} = \begin{pmatrix} 2 & 2 \\ -1 & 3 \end{pmatrix}, \quad \boldsymbol{B} = \begin{pmatrix} -1 & 1 & -2 \\ 1 & 3 & 4 \end{pmatrix}, \quad \boldsymbol{x} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad \boldsymbol{y} = \begin{pmatrix} 1 & 2 & -1 \end{pmatrix},$$

   compute the following. Note that not all operations may be well-defined.

   (a) $\boldsymbol{A} + \boldsymbol{B}$
   (b) $\boldsymbol{AB}$
   (c) $\boldsymbol{BA}$
   (d) $\boldsymbol{Ax}$
   (e) $\boldsymbol{Bx}$
   (f) $\boldsymbol{B}^T\boldsymbol{A}$
   (g) $\boldsymbol{B}^T\boldsymbol{A}^T$
   (h) $\boldsymbol{yB}^T$
   (i) $\boldsymbol{x}^T\boldsymbol{Ax}$
   (j) $\boldsymbol{xB}^T\boldsymbol{y}^T$

2. Prove the following:

   (a) If $\boldsymbol{A}$ and $\boldsymbol{B}$ are symmetric, then $\boldsymbol{AB}$ is symmetric if and only if $\boldsymbol{AB} = \boldsymbol{BA}$. (We then say that "$\boldsymbol{A}$ and $\boldsymbol{B}$ commute".)
   (b) $\boldsymbol{AA}^T$ is symmetric.
   (c) If $\boldsymbol{A}$ is a square matrix, then $\boldsymbol{A} + \boldsymbol{A}^T$ is symmetric.

(3)

3. *Hefferon, p. 54, Exercises 1.12

4. If there exist solutions of the homogeneous system of linear equations other than $\boldsymbol{x} = \mathbf{0}$, express the general solution as a linear combination of linearly independent column vectors:

$$\begin{aligned} x_1 - x_3 &= 0 \\ 3\,x_1 + x_2 + x_3 &= 0 \\ -\,x_1 + x_2 + 2\,x_3 &= 0 \end{aligned} \tag{2}$$

5. Find the general solution of the given set of equations, or else show that there is no solution.

$$\begin{aligned} x_1 - x_2 + x_3 + x_4 &= -1 \\ x_2 + x_3 + 3\,x_4 &= 2 \\ x_1 + 2\,x_3 + 4\,x_4 &= 1 \\ x_2 + x_3 + 3\,x_4 &= 2 \end{aligned} \tag{3}$$

6. (Lial *et al.*, p. 74, Question 49) The U-Drive Rent-A-Truck Co. plans to spend \$6 million on 200 new vehicles. Each van will cost \$20,000, each small truck \$30,000, and each large truck \$50,000. Past experience shows that they need twice as many vans as small trucks. How many of each kind of vehicle can they buy? (2)