# Sets and counting

A <u>set</u> is a collection of objects, called elements

- Write $x \in S$, $S$ a set, if $x$ is an element of $S$
- Sets are written with curly braces:

$$S = \{\text{Alice}, \text{Bob}, \text{Claire}\}$$

$$E = \{n \text{ integers} : n \geq 0,\ n \text{ is even}\}$$ "set of even non-neg. integers"

$$= \{0, 2, 4, 6, \dots\}$$

- A set with no elements is the empty set, $\{\}$ or $\emptyset$

- $A$ is a subset of $B$, $A \subset B$ or $A \subseteq B$, if every element of $A$ is contained in $B$

mean the same!

e.g. $\{A, C\} \subset \{A, B, C\}$; $\{1,2\} \subset \{1,2\}$

$$\{\text{Bob}\} \not\subset \{\text{Alice}, \text{Claire}\}$$

- The union of $A$ and $B$:

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

- The intersection of $A$ and $B$:

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

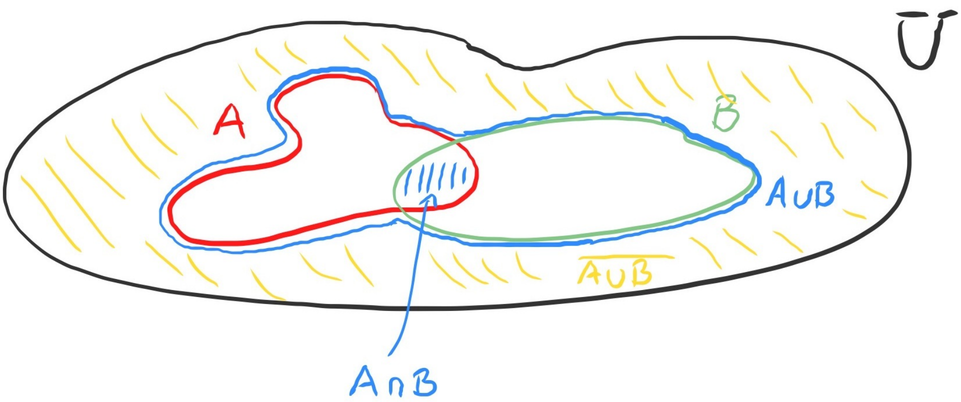
- A universal set $\bar{U}$ is the set of all elements under consideration

- The complement of A:

$$\overline{A} = \{x \in \bar{U} : x \notin A\}$$

- A and B are <u>disjoint</u> if $A \cap B = \emptyset$

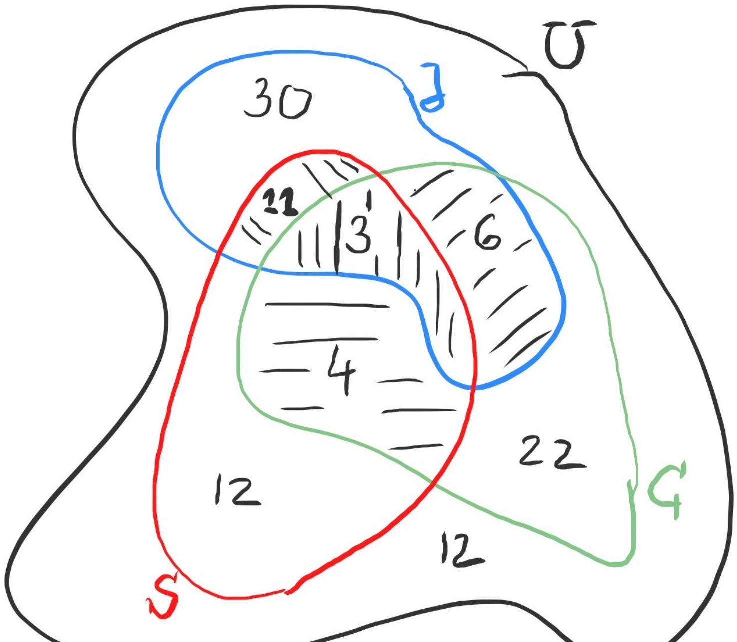
Visualization as "Venn Diagrams"




We can use Venn Diagrams for counting:

<u>Ex</u>: 100 people who exercise

- 50 people jog
- 30 swim
- 35 cycle
- 14 jog & swim
- 7 swim and cycle
- 9 jog & cycle
- 3 do all 3 activities



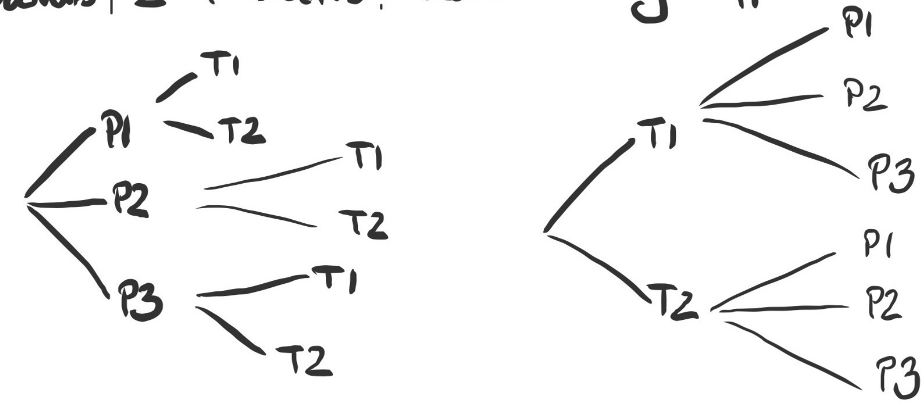

(a) # of people who exercise, but neither jog, swim, or cycle: 12

# $\overline{S \cup J \cup C}$

(b) # of people who do only one of the activities: $12 + 22 + 30 = 64$

Example:

3 pants, 2 T-shirts: how many different outfits are possible




$2 \cdot 3 = 6$ outfits altogether

## Multiplication principle:

Task A can be done in $n$ ways
Task B " " " " $m$ "

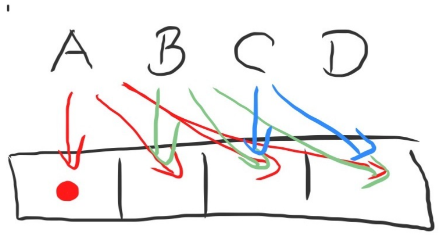
} Completing A and B can be done in $n \cdot m$ ways

## Examples:

① In how many different ways can four question on a True/false test be answered:

$$4 \cdot 2 = 8$$

② In how many different ways can 4 people be seated in a row?




$4 \cdot 3 \cdot 2 \cdot 1 = 24$

$4! = 1 \cdot 2 \cdot 3 \cdot 4$

$n! = 1 \cdot 2 \cdot \ldots \cdot n$

③ How many 3-letter words can be formed from {A, B, C, D, E}
without repetition!    A: 4·3·2·1

A. $5 \cdot 4 \cdot 3 = 60$

These are examples of <u>permutations</u>.

D | B |

In general, these are ordered arrangements of elements from a set where repetitions are not allowed.

<u>Example</u>: How many permutations of the letters of the word ARTICLE have consonants in first and last position?

TARICEL

~~ARTICLE~~

$$\underset{①}{4} \cdot \underset{⑦}{3} \cdot \underset{②}{5} \cdot \underset{③}{4} \cdot \underset{④}{3} \cdot \underset{⑤}{2} \cdot \underset{⑥}{1} = 1440$$

Let $P^n_k$ denote the number of arrangements of a $k$ element subset of an $n$-element set.

So in example ③ above: $n = 5$, $k = 3$

In general: $P^n_k = n(n-1) \cdots (n-k+1)$

$$P^5_3 = 5 \cdot 4 \cdot \underset{(5-3+1)}{\underset{\parallel}{3}} \qquad P^5_2 = 5 \cdot \underset{5-2+1}{\underset{\parallel}{4}}$$

$$P^5_4 = 5 \cdot 4 \cdot 3 \cdot \underset{(5-4+1)}{\underset{\parallel}{2}}$$

Example: In how many distinguishable ways can the letters of the word MISSISSIPPI be re-arranged?

$$\frac{11!}{4!\ 4!\ 2!} = 34\,650$$