Equilibrium configurations of fluids around black holes

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Models of Gravity

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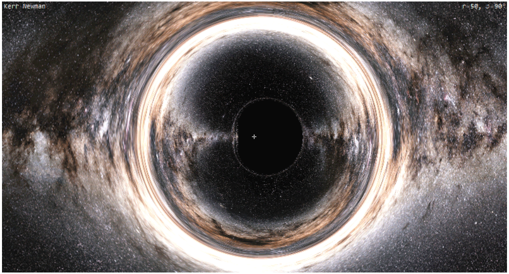
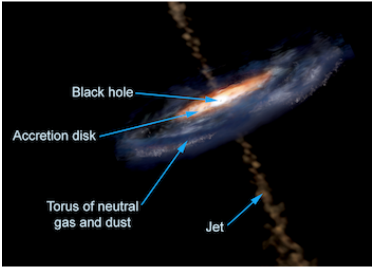
Black holes



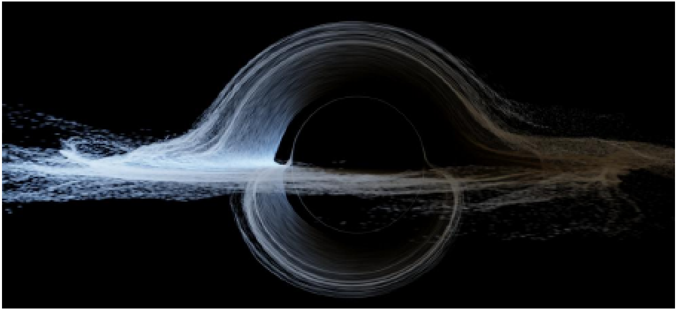
- black holes generate extremely strong gravitational fields
- informations about strong gravitational fields can substantially increase our knowledge about the nature of gravitation
- material accreting onto a compact object can probe deeply into the strong gravity regime

Accretion process



- material orbiting a gravitational object is gradually absorbed
- gravitational and frictional forces compress and raise the temperature of the material
- causes the emission of electromagnetic radiation

The accreted matter



- Cosmic matter mainly exists in the form of plasma
- Different plasma models: hot, cold, collisional, collisionless

→ from infinitely high to very low conductivity;
from fluid to ballistic descriptions

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Black holes

Einstein

- $R_{\mu\nu} + \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$
- $\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu{}_{\nu\rho} \frac{dx^\nu}{d\tau} \frac{dx^\rho}{d\tau} = 0$

Newton

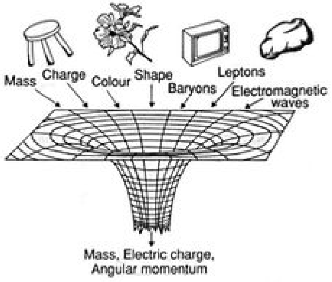
- $\Delta\phi = 4\pi G\rho$
- $\frac{d^2 x^i}{dt^2} + \partial_i \phi = 0$

Most simple solution: Schwarzschild
mass $m = GM/c^2$, spherically symmetric, static, asymptotically flat

$$g = -\left(1 - \frac{2m}{r}\right) c^2 dt^2 + \frac{1}{1 - \frac{2m}{r}} dr^2 + r^2 (\sin^2\theta d\varphi^2 + d\theta^2)$$

$$\left.\begin{array}{l} \blacktriangleright\ \text{coordinate singularity at } r = 2m \\ \blacktriangleright\ \text{curvature singularity ar } r = 0 \end{array}\right\} \text{Black Holes}$$

Black holes



- No–hair theorem/conjecture: isolated black holes are described by mass, angular momentum, and charge only
- In astrophysics charge is usually neglected

- Gravitational force is much smaller than electromagnetic force
- selective accretion of oppositely charged particles from the environment
- net electric charge will quickly decrease to tiny values $Q \lesssim 10^{-18}$
- complete vacuum: pair production $Q \lesssim 10^{-5}$ Eardley+ 1975

(Geometrised units: $Q = \frac{Q_{\rm SI}}{\sqrt{4\pi\epsilon_0 G M}}$)

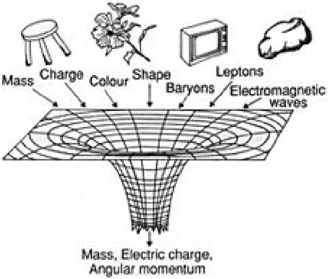
Black hole solution

Kerr-Newman metric & electromagnetic potential

$$
\begin{aligned}
g = &\frac{\Delta}{\rho^2}(dt - a\sin^2\theta d\phi)^2 - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2 \\
&- \frac{\sin^2\theta}{\rho^2}(adt - (r^2+a^2)^2 d\phi)^2 \\
A = &\frac{Qr}{\rho^2}(dt - a\sin^2\theta d\varphi)
\end{aligned}
$$

- mass $m = \frac{GM}{c^2}$, rotation $a = \frac{J}{Mc}$, electric charge $Q^2 = \frac{Q_{\mathrm{SI}}^2 G}{4\pi\epsilon_0 c^4}$
- $\Delta = r^2 - 2mr + a^2 + Q^2$, $\rho^2 = r^2 + a^2\cos^2\theta$.
- stationary, axially symmetric, asymptotically flat

Charged black holes?



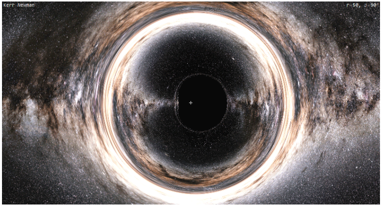
- black holes are usually not isolated
- rotation of a black hole interacts with electromagnetic fields

- magnetic fields from accretion disk, galactic field, ...
- interaction $\rightarrow$ selective accretion of charged particles
- the resulting charge is proportional to the magnetic field strength
- example: asymptotically uniform magnetic field $Q = 2aB$ Wald 1974

Charged black holes?

Astrophysical black holes

- most likely have nonvanishing but very tiny charge to mass ratios
- many orders of magnitude below unity



Estimate for charge of Sgr A* Zajacek+ 2018

- based on observation of bremsstrahlung
- results in: $Q_{\mathrm{SI}} \lesssim 3 \times 10^{8}\mathrm{C}$ or $Q \lesssim 4 \times 10^{-19}$

Astrophysical charged black holes

- very tiny charge to mass ratio

$\rightarrow$ influence on spacetime geometry negligible small

- electromagnetic interaction is much stronger than gravitational force
- is there an influence on charged particles?

Order of magnitude estimate

- Kerr-Newman black hole with $Q = 10^{-19}$
- equations of motion contain product qQ
- for free electrons: $q = \frac{q_{\mathrm{SI}}}{\sqrt{4\pi\epsilon_0 G}\mu_e} \approx -2 \times 10^{21}$
- for free protons: $q \approx 1 \times 10^{18}$
- then $qQ \approx 10^{-1} - 10^{2}$ $\quad \rightarrow$ a priori not negligible

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Models of accretion

- General relativistic magneto-hydrodynamics (GRMHD) simulations
- Computationally expensive: range of scales, dimensions, turbulence, radiation, ...
- Analytical models to understand the general relevant physical processes
- Also serve as test beds and initial conditions for simulations

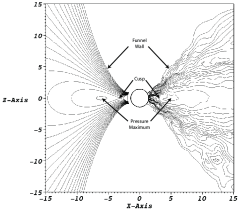
Analytical relativistic models

- Spherical accretion models
- Accretion disks (thick, thin, slim, adaf)

Thick accretion disks

Astrophysical motivation

- Active galactic nuclei have a very high accretion rate ($\sim$ critical Eddington rate)
- Close to the central black hole, disks with high accretion rates have to be geometrically thick

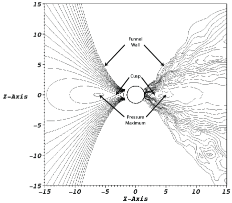


Models of thick disks (Polish doughnuts)

- original model in the Kerr background Abramowicz+ 1978
- additional toroidal magnetic field attached to the fluid Komissarov 2006
- charged fluid in Reissner-Nordström background Kovar+ 2011
- charged fluid & em-field, static spacetimes Kovar+ 2014, 2016

$\rightarrow$ charged fluids in Kerr background + em-fields?

Original model



Geometrically thick accretion disks

- fluid in hydrostatic equilibrium close to a black hole
- very low viscosity

Assumptions of the model

- The whole setup is stationary and axially symmetric
- reflection symmetry to equatorial plane
- matter described by perfect fluid, polytropic equation of state
- the fluid motion is purely circular ($\rightarrow$ no accretion!)
- test-fluid: does not influence the spacetime

Gravitation and pressure form equilibrium configurations

Basic relations

Basic equation

- energy conservation: $\nabla_\mu T^{\mu\nu} = 0$

$\rightarrow$ Euler equation: $(\epsilon + p)\dot{U}_\alpha + h^\mu_\alpha \partial_\mu p = 0$

Circular motion

- angular velocity $\omega = U^\varphi / U^t$, angular momentum $l = -U_\varphi / U_t$
- then $U^\mu = U^t(\delta^\mu_t + \omega\delta^\mu_\varphi)$
- from $g_{\mu\nu}U^\mu U^\nu = -1$:

$$(U^t)^2 = -1/(g_{tt} + 2\omega g_{t\varphi} + \omega^2 g_{\varphi\varphi})$$

- this gives

$$\dot{U}_\alpha = -\partial_\alpha(\ln U^t) + \frac{l\partial_\alpha\omega}{1 - \omega l}$$

Final equations

- insert in Euler equation:

$$\frac{\partial_\mu p}{p+\epsilon} = \partial_\mu(\ln U^t) - \frac{l\partial_\mu \omega}{1-\omega l}$$

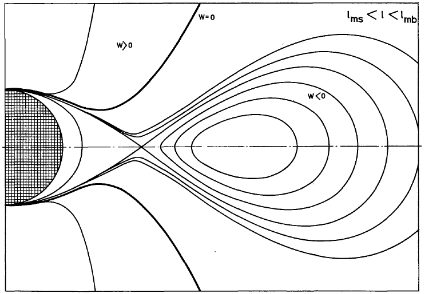
- Von-Zeipel: integrable iff ($l = \mathrm{const}$ *or* $\omega = \mathrm{const}$ *or* $l = l(\omega)$)
- usually $l = \mathrm{const}$ is used
- The effective potential W is defined as

$$-W = \int_x \frac{\partial_\mu p}{p+\epsilon} dx^\mu = \int_p \frac{dp}{p+\epsilon(p)}$$

- bound structures exist if pressure has local maximum
- extrema mark the center and the cusp

Example: Schwarzschild equipressure surfaces

- $l = \mathrm{const}$ is assumed
- (bound fluid structures impossible for $\omega = \mathrm{const}$)



from Abramowicz et al 1977

- center and cusp on Keplerian/geodesic orbits
- center $\approx r_{\mathrm{ms}}$, cusp $\approx r_{\mathrm{mb}}$

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Basic assumptions

- start with same assumptions as in original model
- add an em-field: $(A_\mu) = (A_t, A_\varphi, 0, 0)$
- matter is charged perfect fluid with $F^{\mu\nu}_{\mathrm{INT}} \ll F^{\mu\nu}_{\mathrm{EXT}}$

Maxwell's equations

$$\nabla_{[\mu} F_{\nu\rho]} = 0\,, \quad \nabla_\mu F^{\mu\nu} = \mu_0 j^\nu = \mu_0(\rho_q U^\nu + \sigma F^{\nu\alpha} U_\alpha)$$

Ideal MHD

- $\sigma \to \infty$: $F^{\nu\alpha} U_\alpha = 0$ implies $\partial_\mu A_t + \omega \partial_\mu A_\varphi = 0$
- implies $\omega = \mathrm{const}$ or $A_\varphi = A_\varphi(\omega)$ Bonazzola+ 1993
- $\omega = \mathrm{const}$ gives $A_t = -\omega A_\varphi$

$\rightarrow$ assume a finite conductivity

Pressure equation

Energy-momentum conservation and Maxwell's equations result in

$$\frac{\partial_\mu p}{p+\epsilon} = \partial_\mu \ln U^t - \frac{l\partial_\mu \omega}{1-\omega l} + \frac{\rho_q}{p+\epsilon} F_{\mu\alpha} U^\alpha + \frac{1}{p+\epsilon} \sigma h^\nu_\mu F_{\nu\alpha} F^{\alpha\beta} U_\beta$$

stationarity and axial symmetry: $\partial_t p = 0$, $\partial_\varphi p = 0$

- conductivity term has to vanish for $\mu = t, \varphi \quad \rightarrow \sigma = 0$

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integrability condition $\partial_\mu \partial_\nu p = \partial_\nu \partial_\mu p$:

- EOS and von Zeipel theorem
- find scalar function S such that $\frac{\rho_q}{p+\epsilon} F_{\mu\alpha} U^\alpha = f(S) \partial_\mu S$

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Bound structures exist if pressure has local maximum

Construction of bound fluid structures

Effective potential $h = \int_0^p \frac{d\tilde{p}}{\tilde{p}+\epsilon(\tilde{p})}$

- From h we can derive the physical characteristics p, ϵ, ρ_q (given a specific equation of state)
- Necessary and sufficient conditions

$$0 = \partial_\mu h(r_c, \theta_c) \quad \text{for} \quad \mu = r, \theta$$
$$0 > \det H(r_c, \theta_c) \quad \text{and} \quad 0 > \partial^2_{rr} h(r_c, \theta_c)$$

- restrict to equatorial plane and axis of symmetry
- $\partial_r h(r_c, \theta_c) = 0$ gives normalisation for charge distribution $f(S)$
- $\partial^2_{rr} h(r_c, \theta_c) < 0$ restricts choice of ω and/or l
- $\partial^2_{\theta\theta} h(r_c, \theta_c) < 0$ restricts form of $f(S)$

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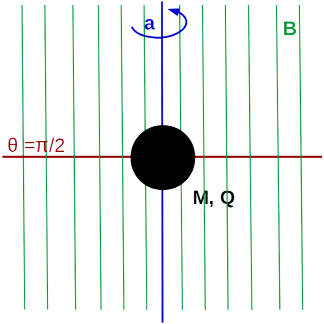
Rigid rotation



- Rotating black hole $\rightarrow$ determines gravitational field
- asymptotically uniform magnetic test field, aligned with rotation axis

$\rightarrow$ induces test electric field of black hole $Q \sim B$

- assume rigid rotation $\omega = \mathrm{const} \rightarrow$ impossible for uncharged case!
- scalar function $S = A_t + \omega A_\varphi$, $f(S) = \frac{\rho_q}{p+\epsilon} U^t = kS^n$
- Found equilibrium configurations on the equator and the polar axis

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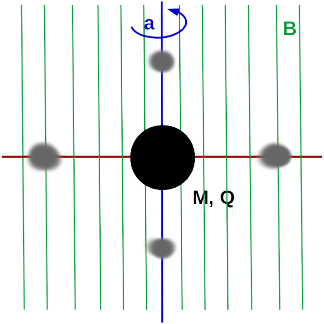
Rigid rotation



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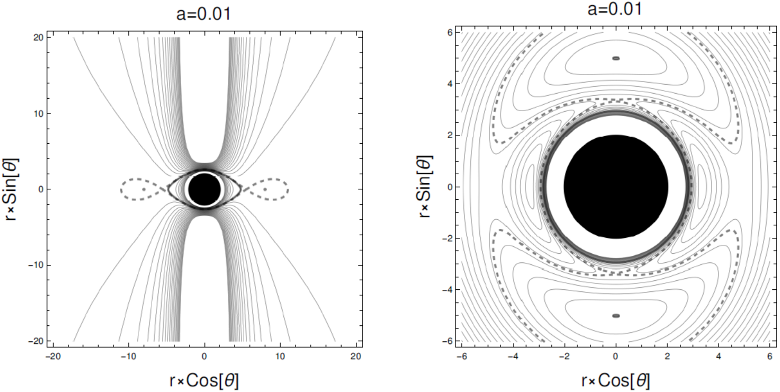
- Found equilibrium configurations on the equator and the polar axis

Main results

- charged equilibrium configurations can be in rigid rotation
- the rotation has a major impact on the domain of existence of bound structures as compared to the non-rotating case
- equilibrium structures move away from black hole for increasing rotation
- morphology changes: from prolate to oblate
- rotating black holes support rigidly rotating bound structures on the axis of symmetry: polar clouds

Example equilibrium configurations



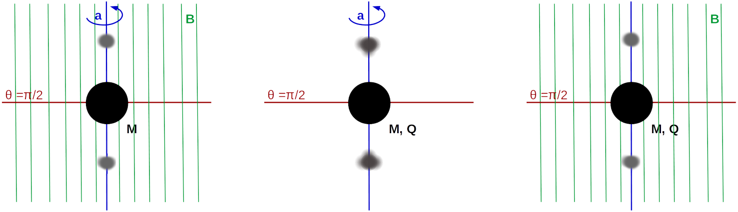
Equipotential surfaces of the pressure.
Left: equatorial torus ($f(S) \sim S^{-2}$), right: polar cloud ($f(S) \sim S$).

Special cases

We also considered two special cases:

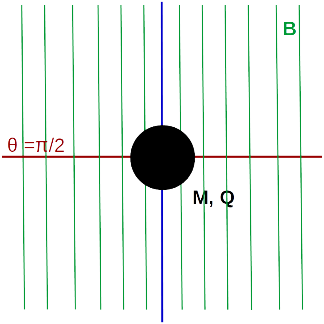
- $Q = 0$, $B \neq 0$
- $Q \neq 0$, $B = 0$

In both cases, we found polar cloud configurations:
this is impossible for non-rotating black holes



Kovar+ 2014

Constant angular momentum

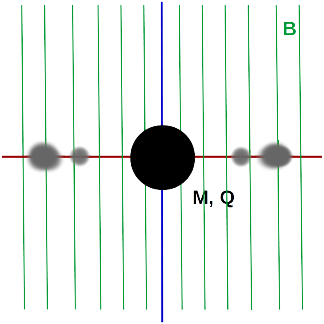


Slightly different scenario:

- non-rotating black hole
- assume $l = \mathrm{const}$ instead of $\omega = \mathrm{const}$

- scalar function $\partial_\mu S = \partial_\mu A_t - l g^{\varphi\varphi} \partial_\mu A_\varphi$, $f(S) = \frac{\rho_q}{p+\epsilon} U_t = k S^n$
- found (double) tori on the equator

Constant angular momentum



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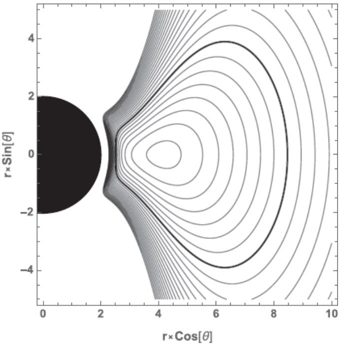
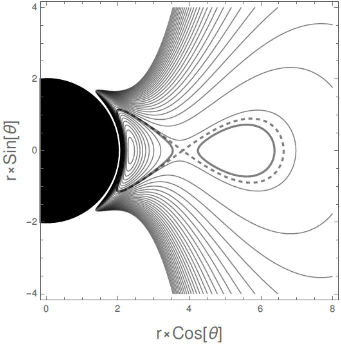
Possible configurations

Assuming $f(S) \sim S$:

- structures with a cusp only (no bound structures)
- bound tori without a cusp (no overflow $\rightarrow$ no accretion)
- bound structures with inner and outer cusps (in- and outflow possible)
  generally the cusps are on different equipotential surfaces
- two bound structures connected by a cusp

Bound and double tori





Equipotential surfaces of the pressure

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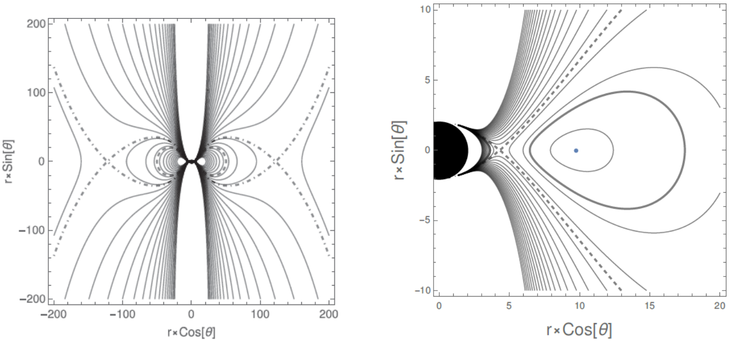
Double cusp



Equipotential surfaces of the pressure

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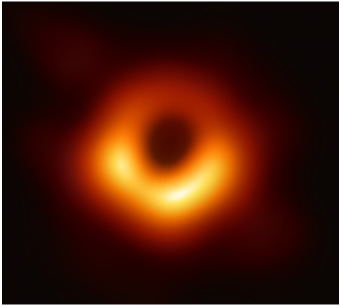
Summary



Astrophysical black holes

- most likely have a very small electric charge $Q < 10^{-18}$
- influence on spacetimes geometry negligible
- influence on charged matter a priori non-negligible

Summary

- general construction method for charged perfect fluids in axially symmetric and stationary spacetimes and em-fields
- uniform magnetic field and electric charge of the black hole
- rigid rotation or constant angular momentum

Rigid rotation

- equilibrium configurations in equatorial plane and on polar axis
- pressure maxima shift to larger radii for increasing rotation
- polar clouds need at least two nonvanishing parameters in (a, Q, B)

Constant angular momentum

- double cusps and double tori possible

Discussion and outlook

We discussed the influence of charge in (semi-)analytical toy models

- equilibrium configurations $\leftrightarrow$ dynamical flow
- vanishing conductivity $\leftrightarrow$ ideal MHD

$\rightarrow$ these caveats could make the charge effects vanish!

$\rightarrow$ simulations with small conductivity?

Open questions in the construction method

- stability analysis
- preferred charge distributions / zero net charge possible?
- off-equatorial / off-polar structures