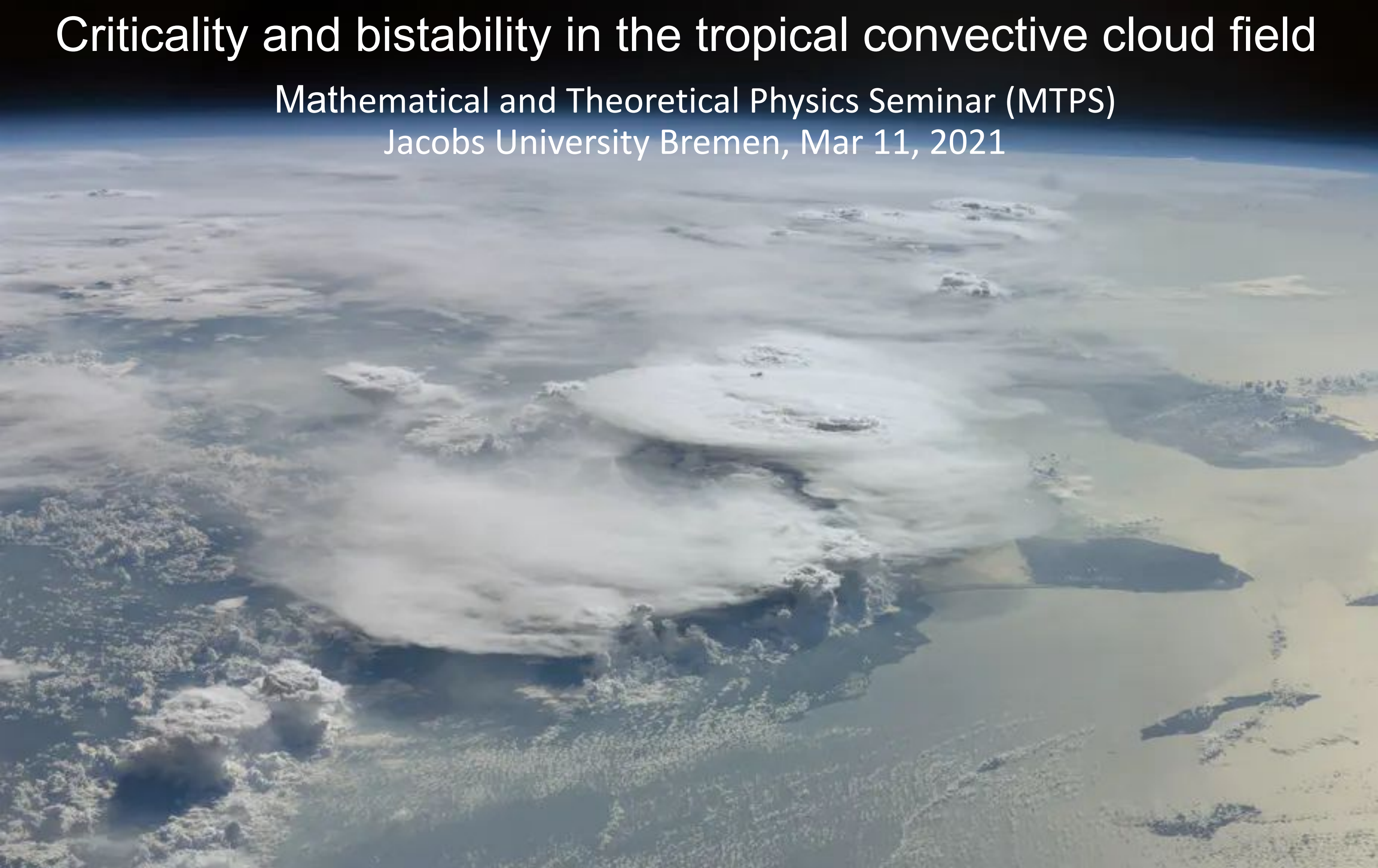
Criticality and bistability in the tropical convective cloud field

Mathematical and Theoretical Physics Seminar (MTPS)
Jacobs University Bremen, Mar 11, 2021



Jan O. Haerter

Jacobs University Bremen and Niels Bohr Institute
Leibniz ZMT

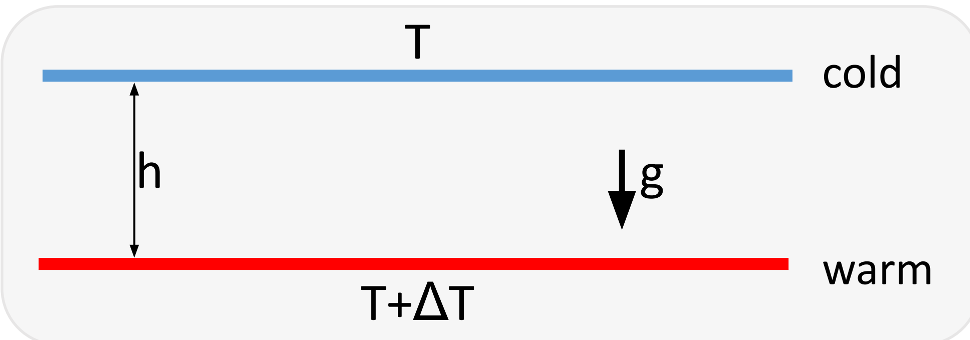
European Research Council

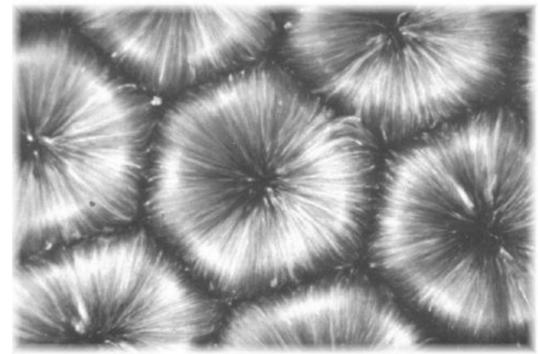
VILLUM FONDEN

- What is (moist) convection?
- Cold pools – Drivers of convective organization
- Convective self-aggregation
- Cloud interactions: an explanation for self-aggregation
- Mapping out the phase diagram
- Conclusion

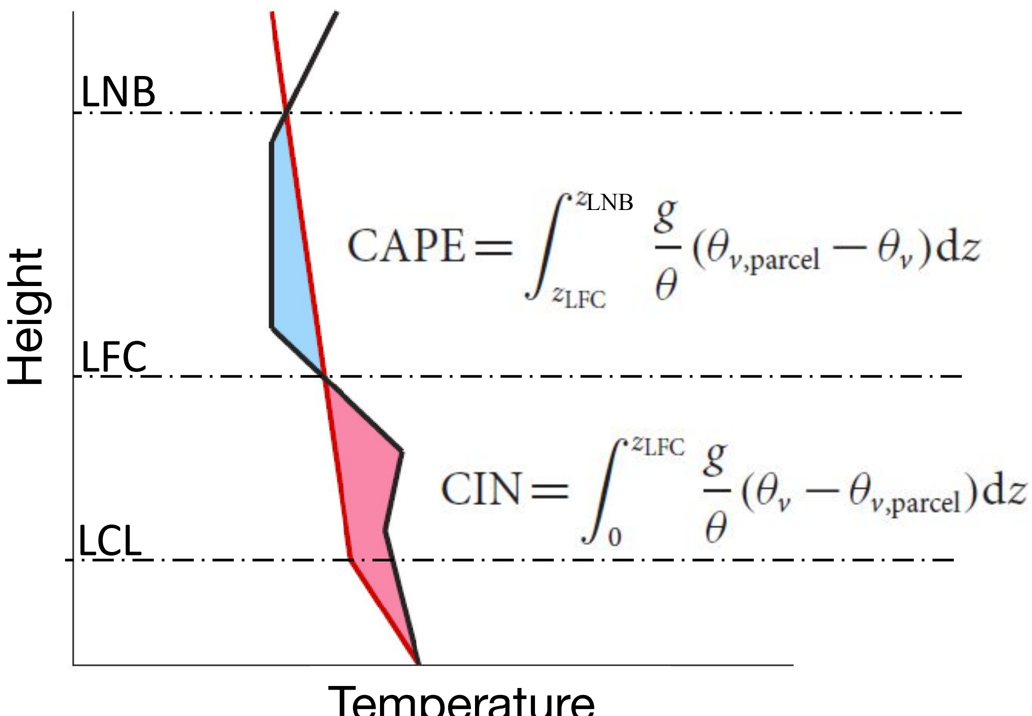
What is (moist) convection?

$$\frac{\text{Bouyancy force}}{\text{Viscous force}} \sim \frac{g\rho}{\nu U/h^2} \sim \frac{g\alpha\Delta T}{\nu\kappa/h^3} = \frac{\alpha\Delta T g h^3}{\nu\kappa} = \mathrm{Ra}$$



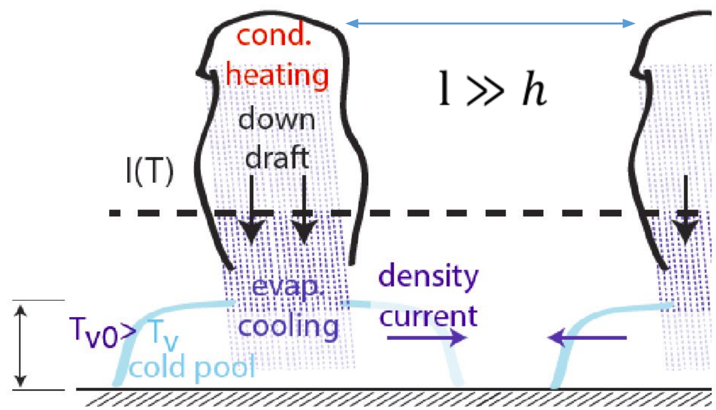


M. Van Dyke (1982)



$$\mathrm{CAPE} = \int_{z_{\mathrm{LFC}}}^{z_{\mathrm{LNB}}} \frac{g}{\theta}(\theta_{v,\mathrm{parcel}} - \theta_v)\mathrm{d}z$$

$$\mathrm{CIN} = \int_0^{z_{\mathrm{LFC}}} \frac{g}{\theta}(\theta_v - \theta_{v,\mathrm{parcel}})\mathrm{d}z$$



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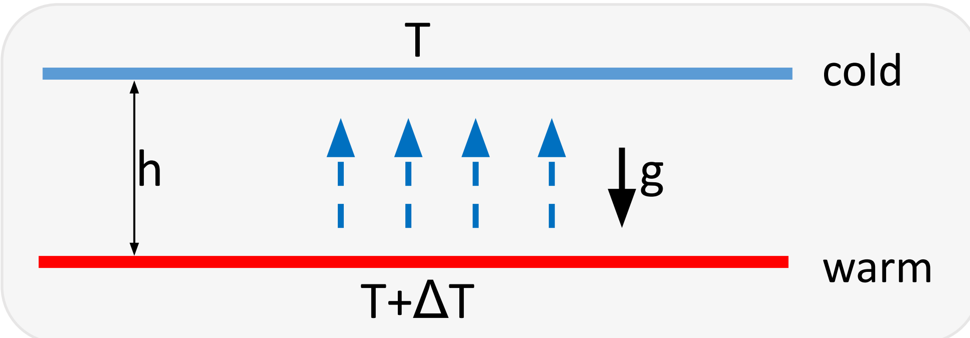

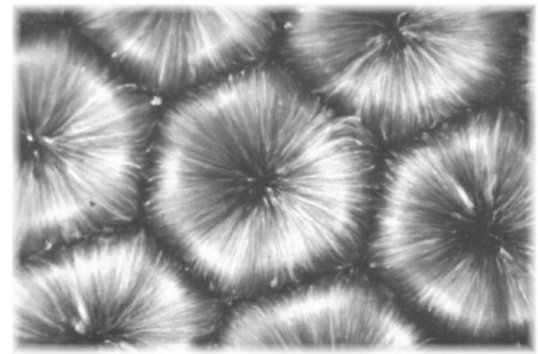

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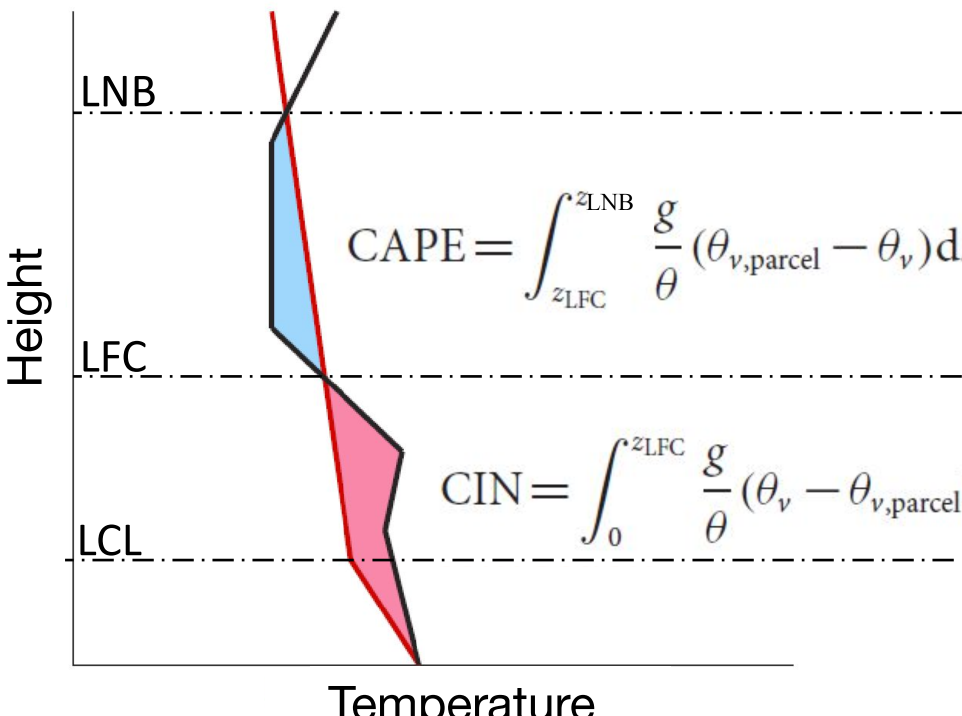

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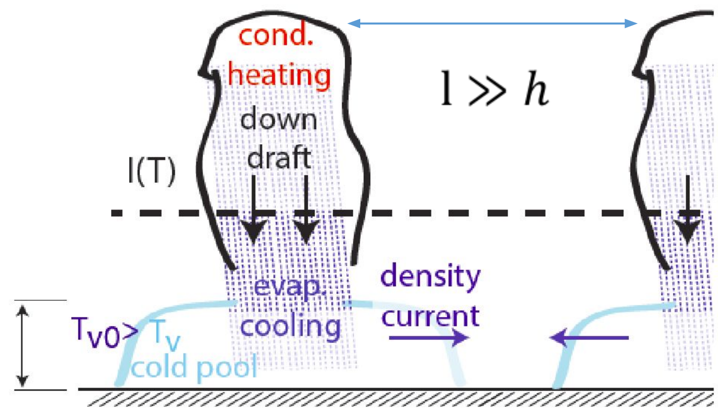

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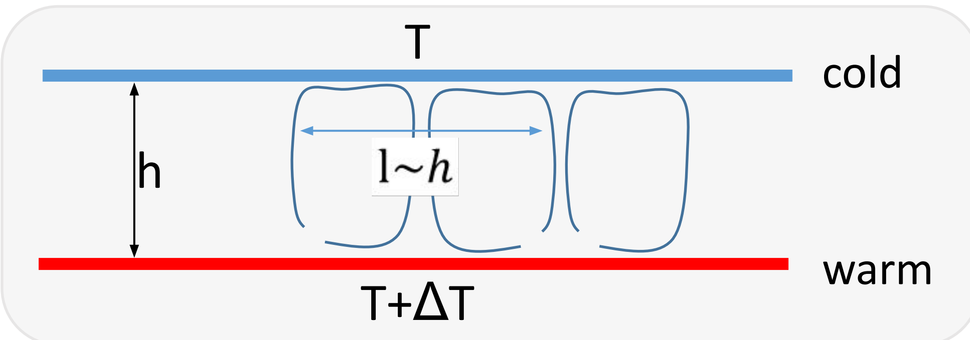

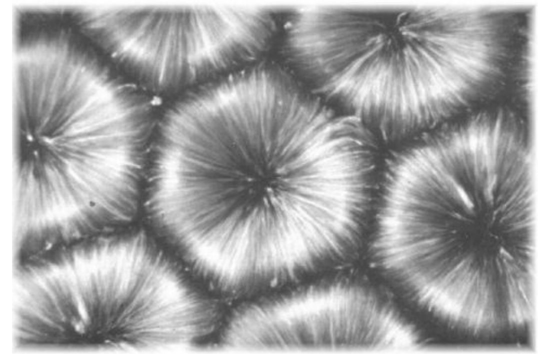

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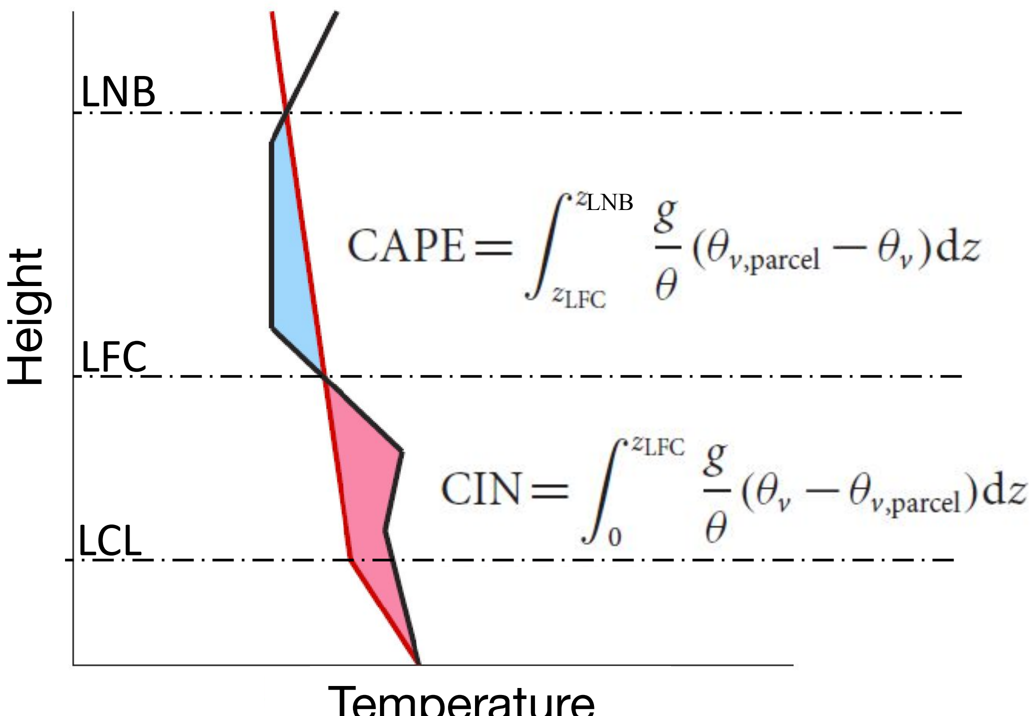

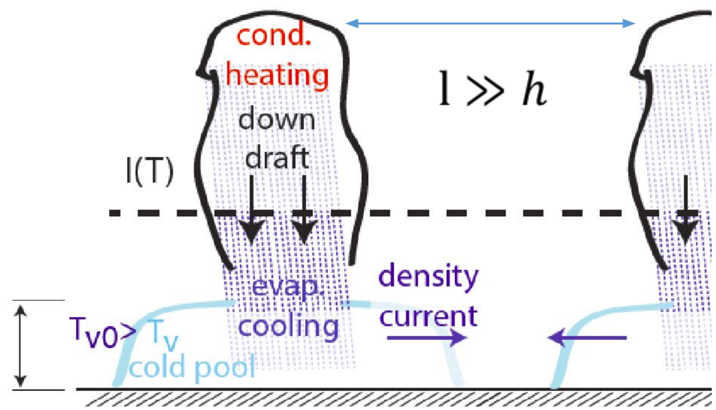

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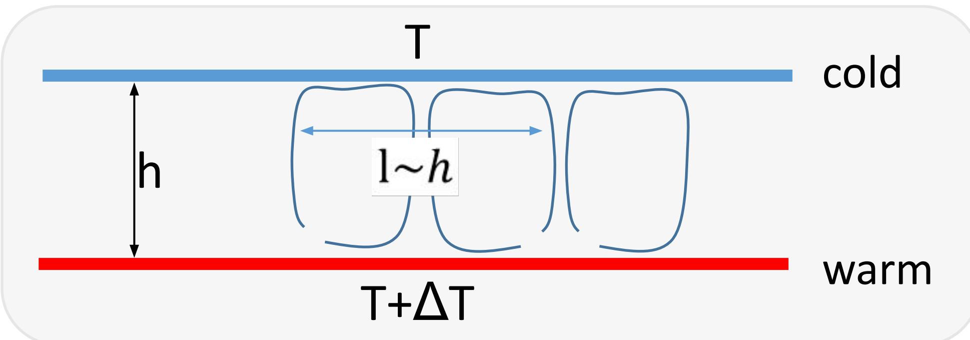

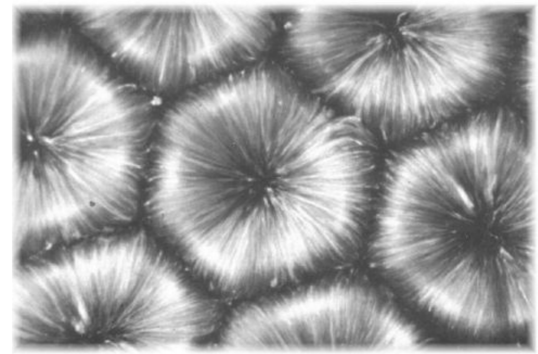

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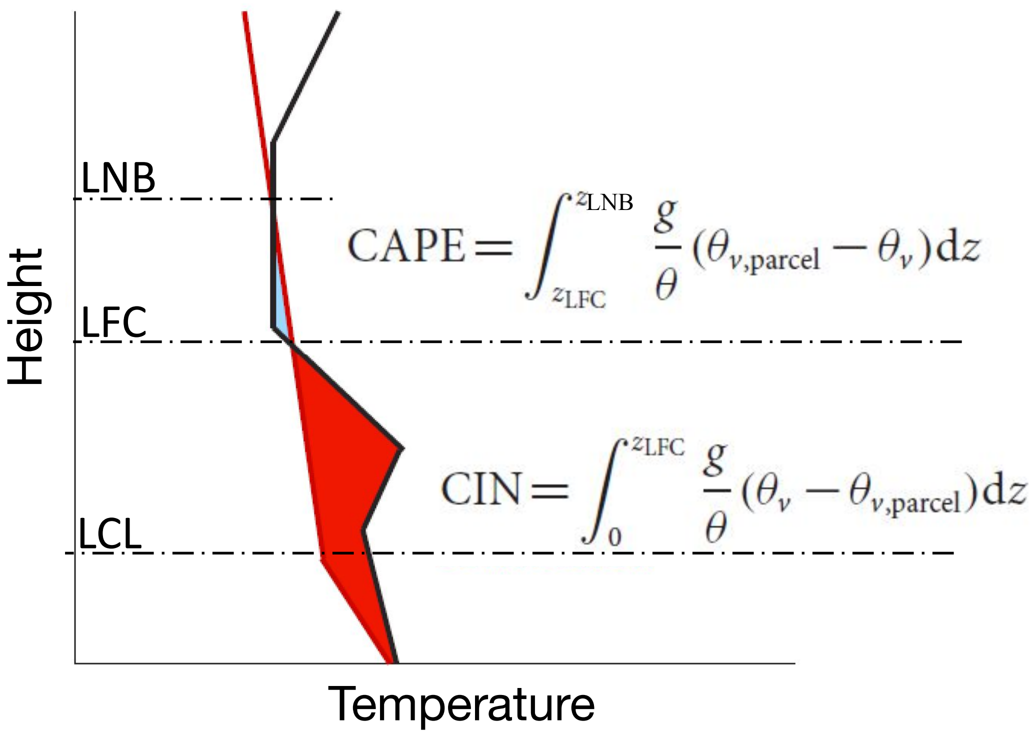

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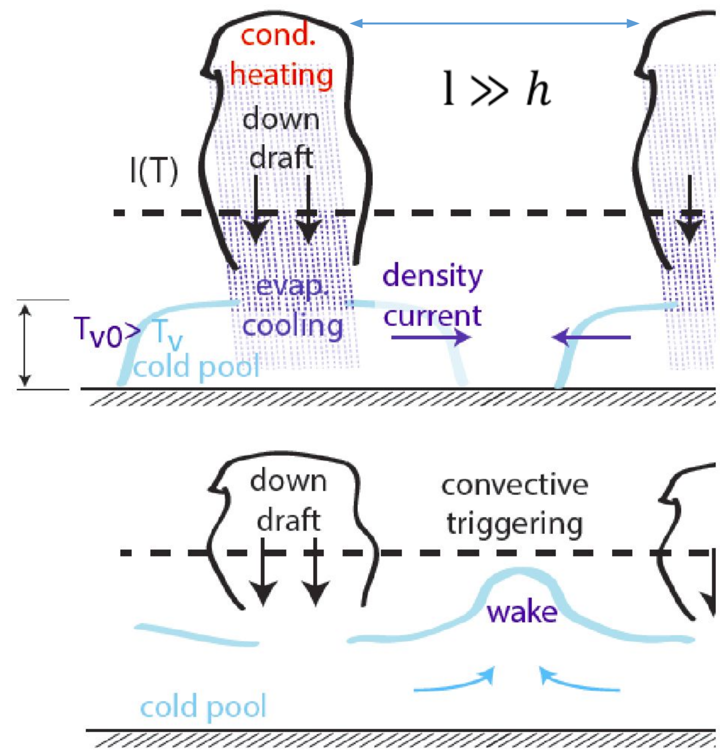
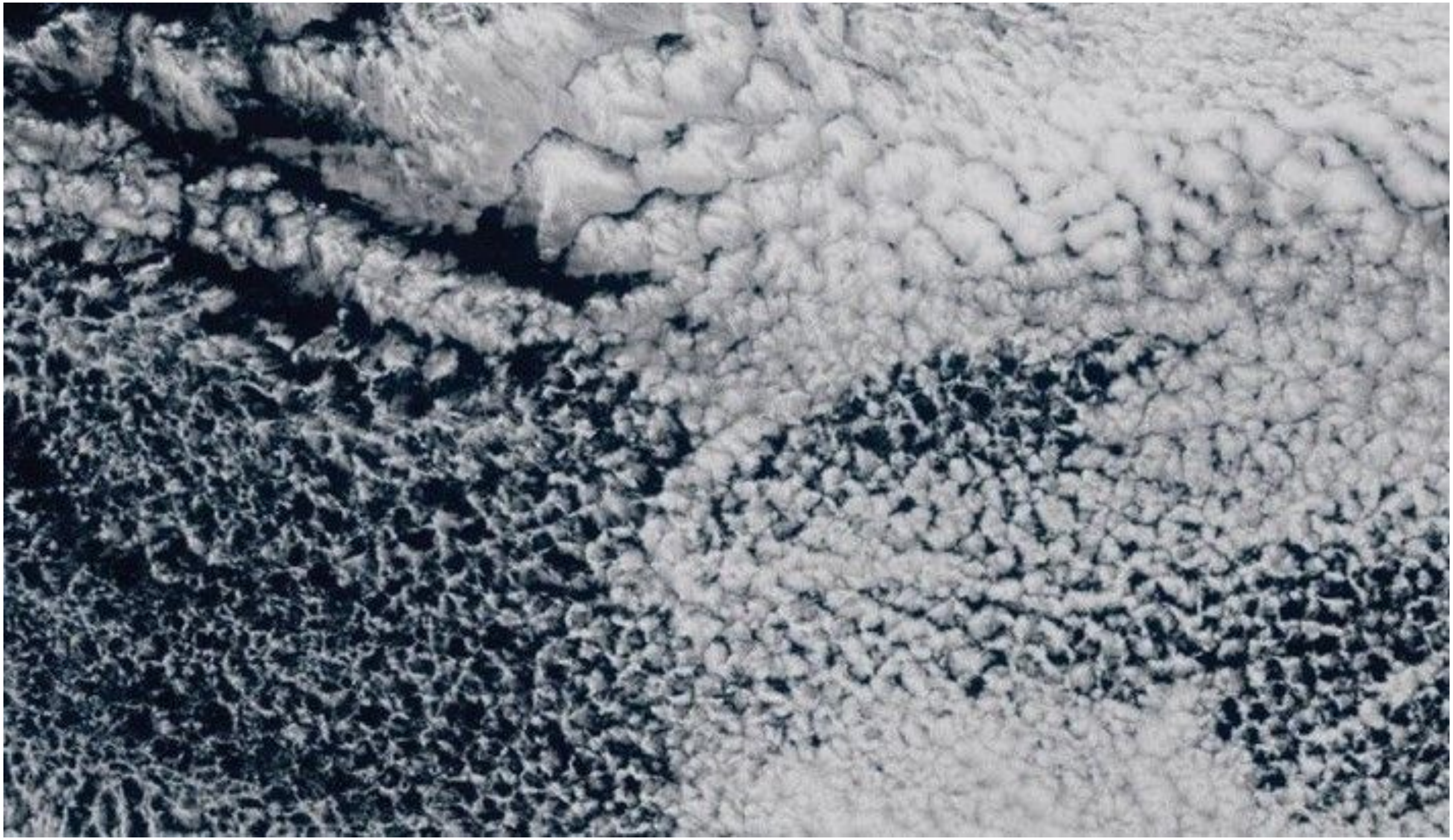


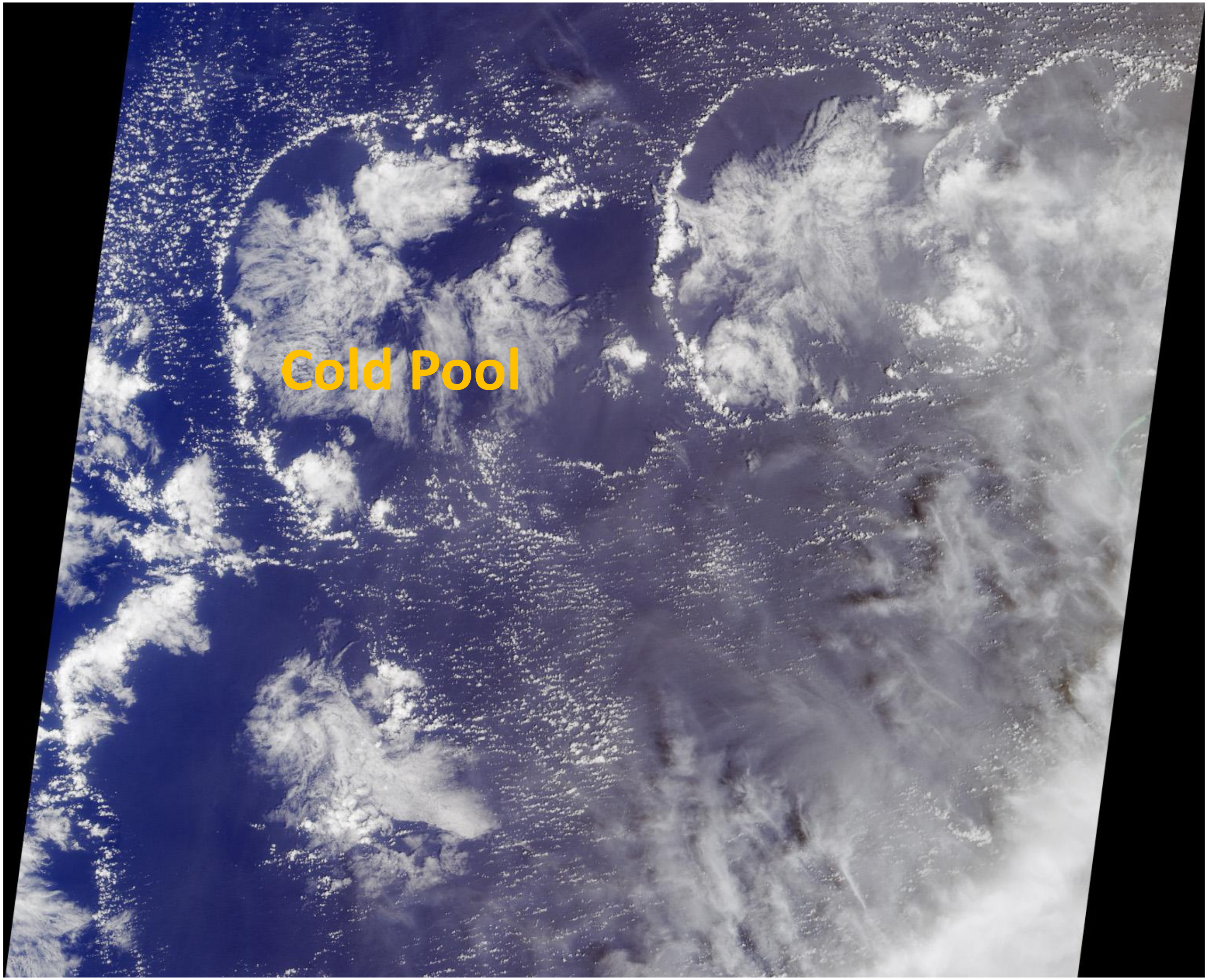


weakly precipitating marine stratocumuli

Cold Pools – Drivers of Organization

$\approx 10\ km$

Cold Pools – Drivers of Organization

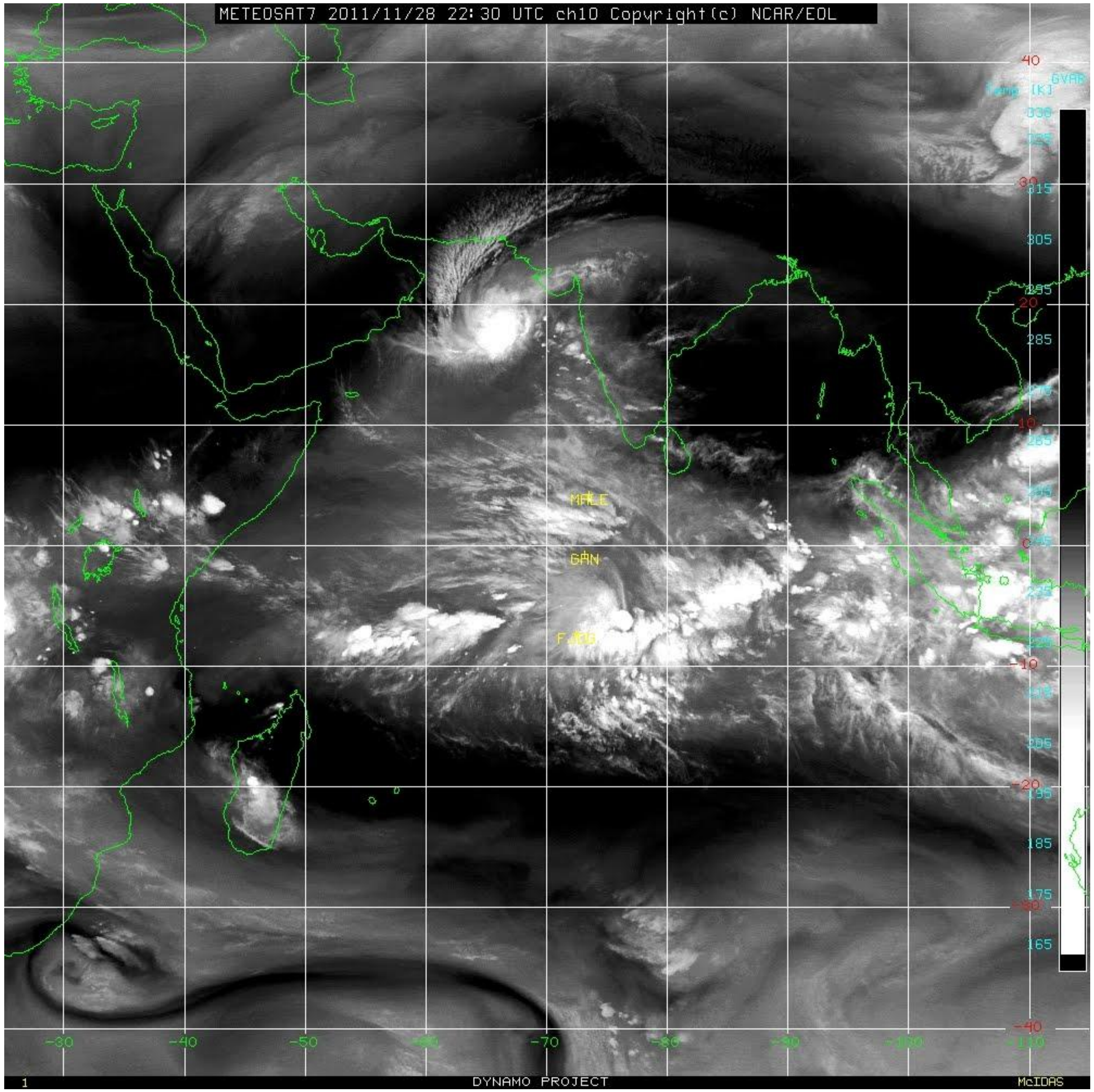


near Barbados

Zuidema et al., 2015

$\approx 200\ km$

Large-scale clustering of convection

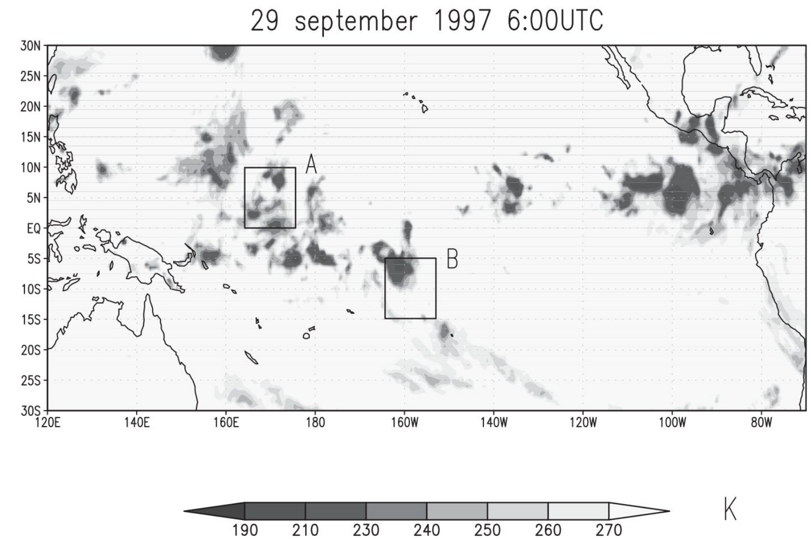


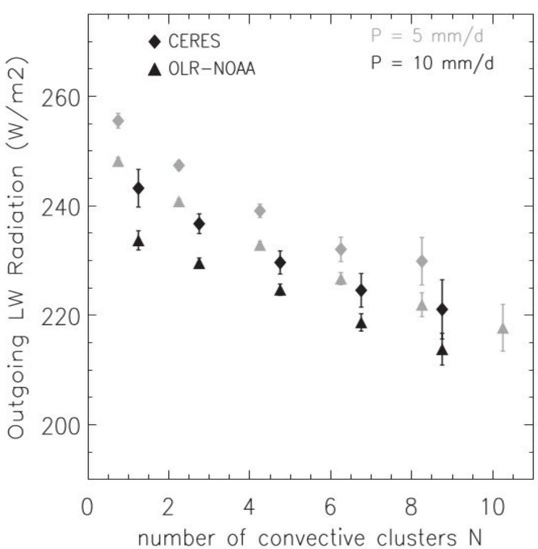
$\approx 10{,}000\ km$

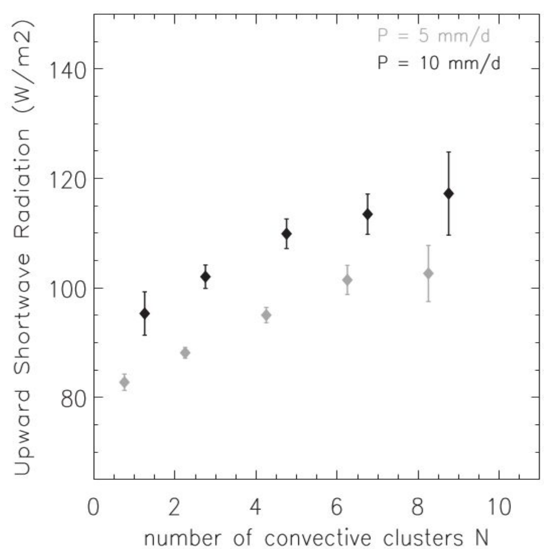
Observing Convective Clustering

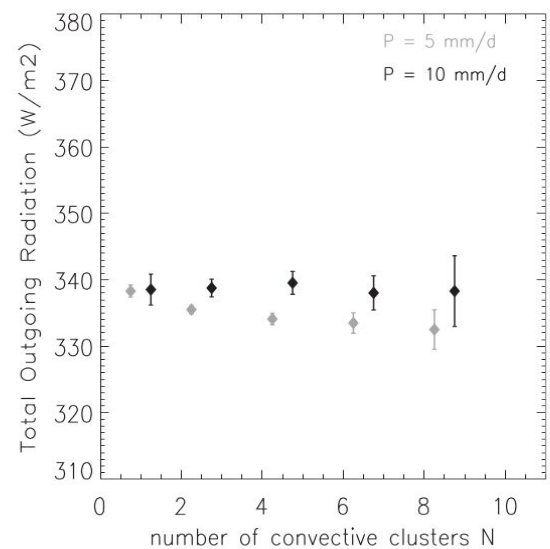
- Defining cloud clusters
- Conditioning on rainfall
- Measuring OLR

More clustering - more OLR



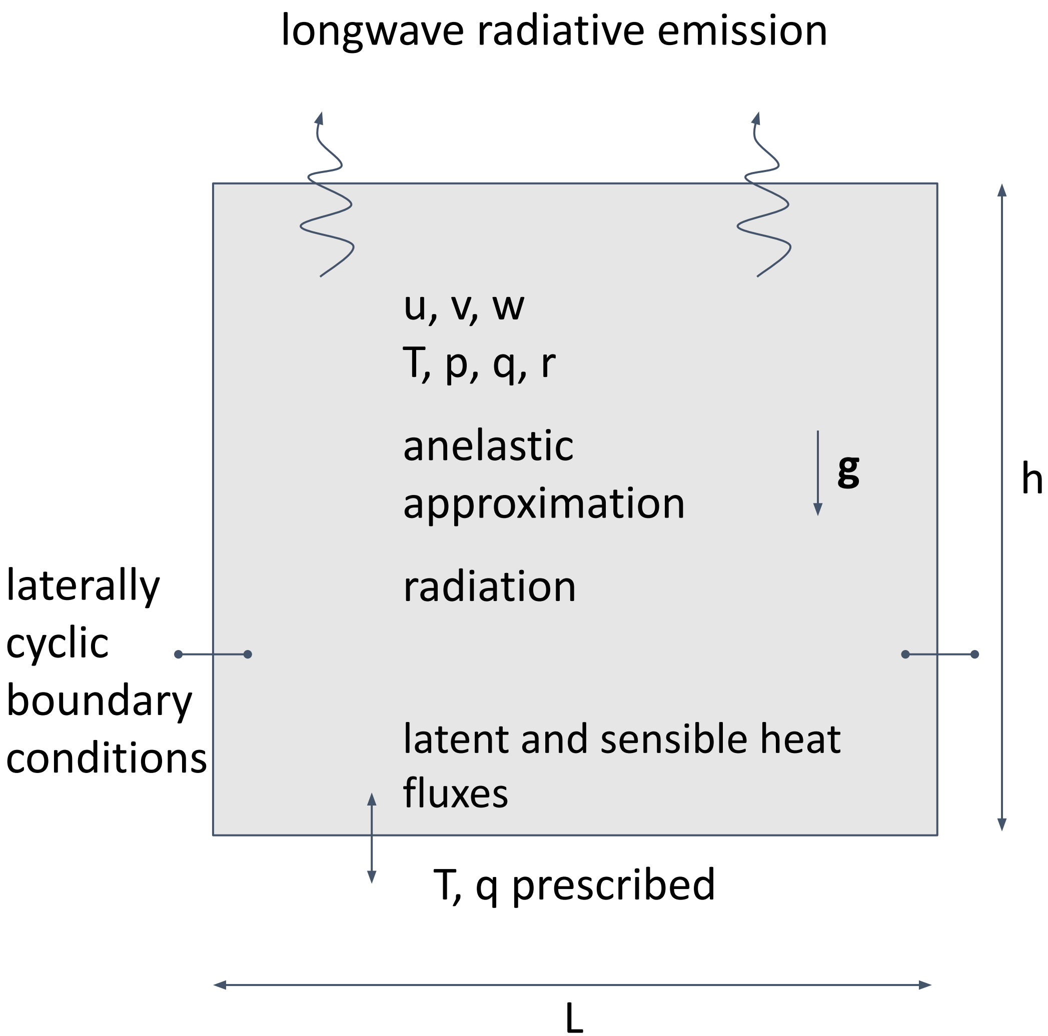






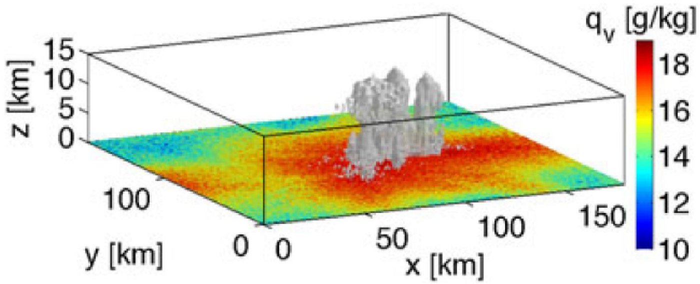
Tobin et al., 2012

Large-eddy simulations



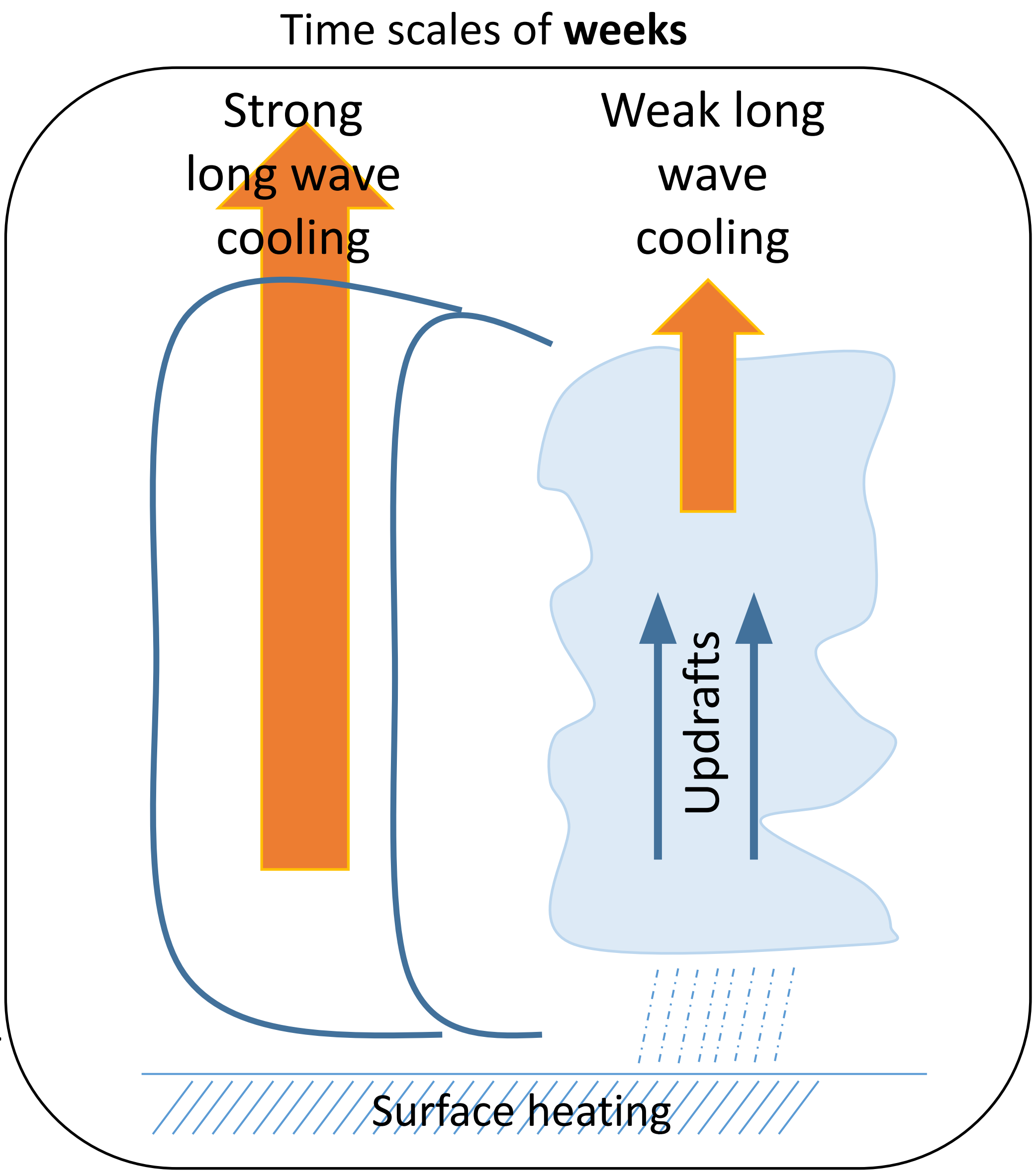
- no rotation (Coriolis=0)
- no annual cycle
- *often*: constant boundary conditions
- horizontal resolution ≃ 500-1000 m
- vertical resolution ≃ 100 m
- domain size: L ≃ 1000 km
- uniform initial conditions
- often: run to radiative convective equilibrium

Convective Self-Aggregation (CSA)



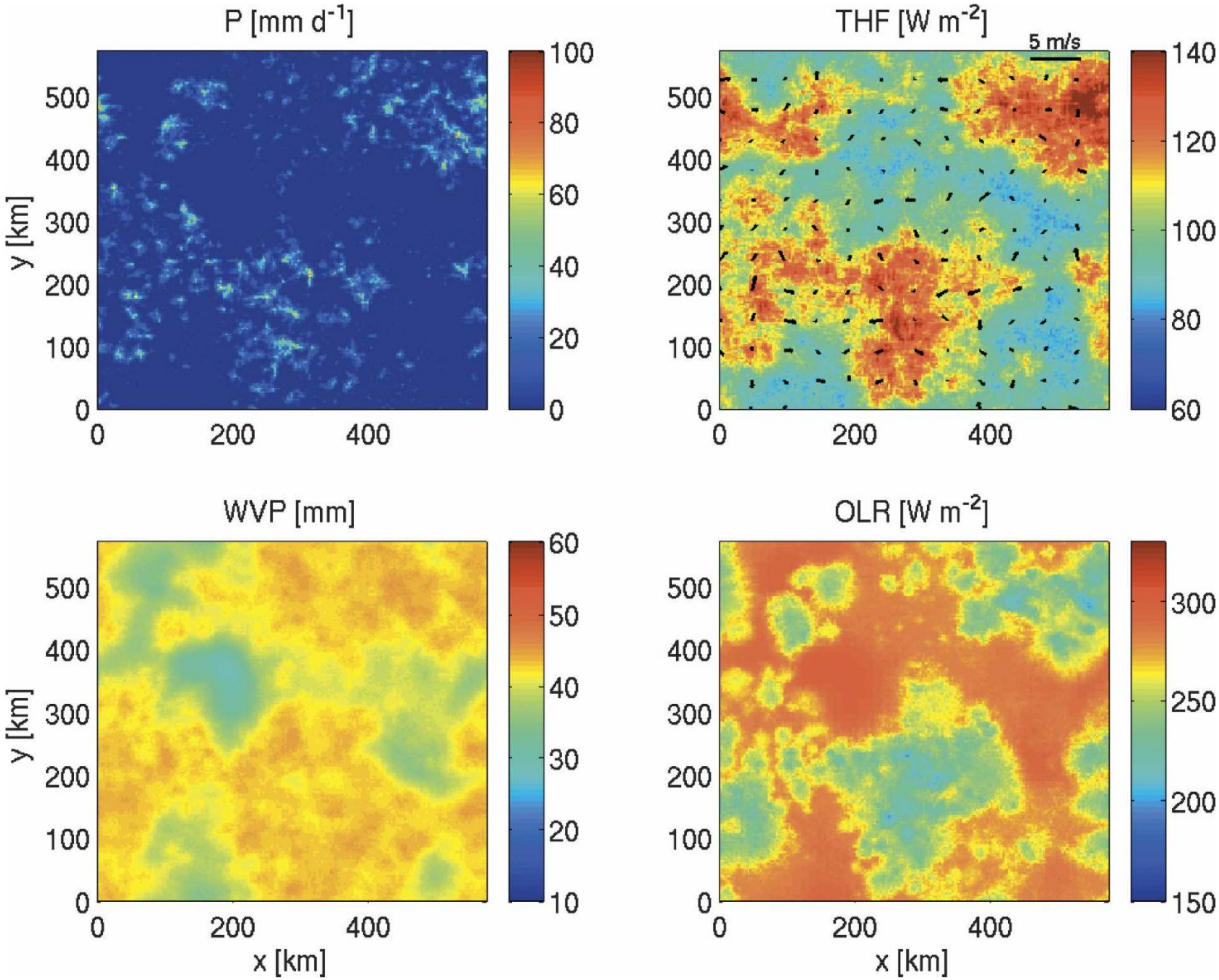
Muller & Held, 2012

vast literature on self-aggregation:
Emanuel, Bretherton, Bony, Jeevanjee & Romps, Hohenegger & Stevens, Craig, Muller, Soden, etc.



Simulated CSA

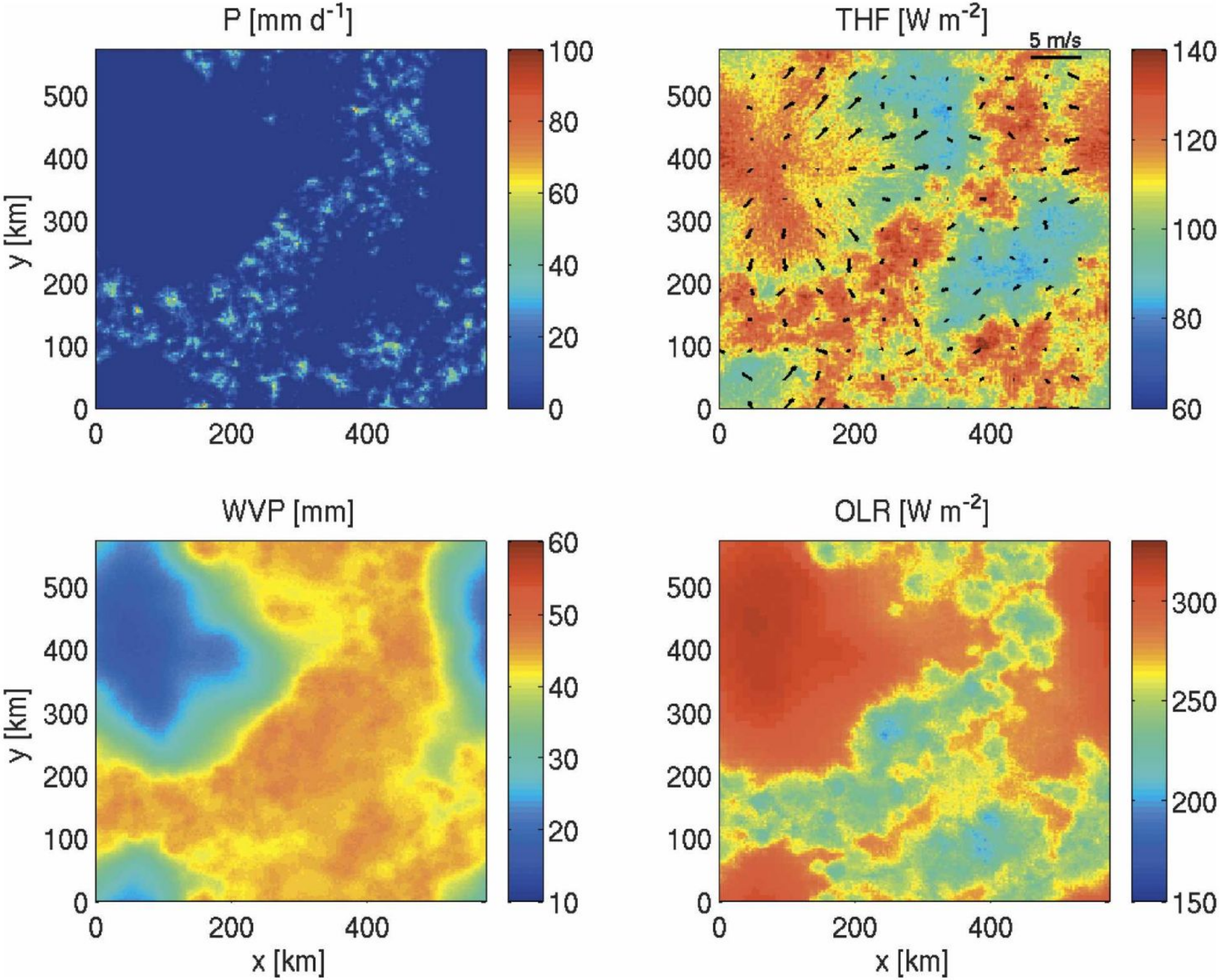
day 10



Bretherton et al., 2005

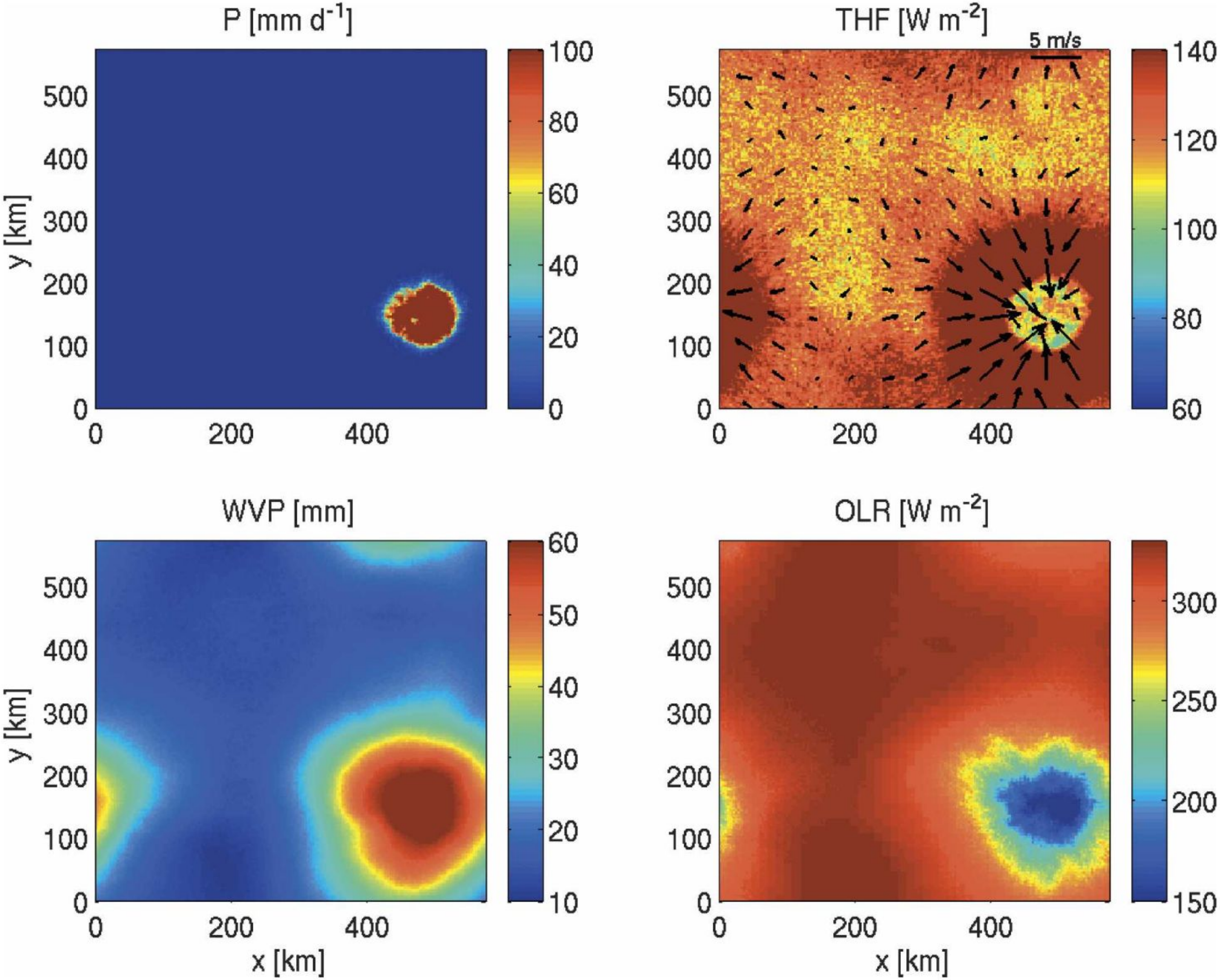
Simulated CSA

day 20



Bretherton et al., 2005

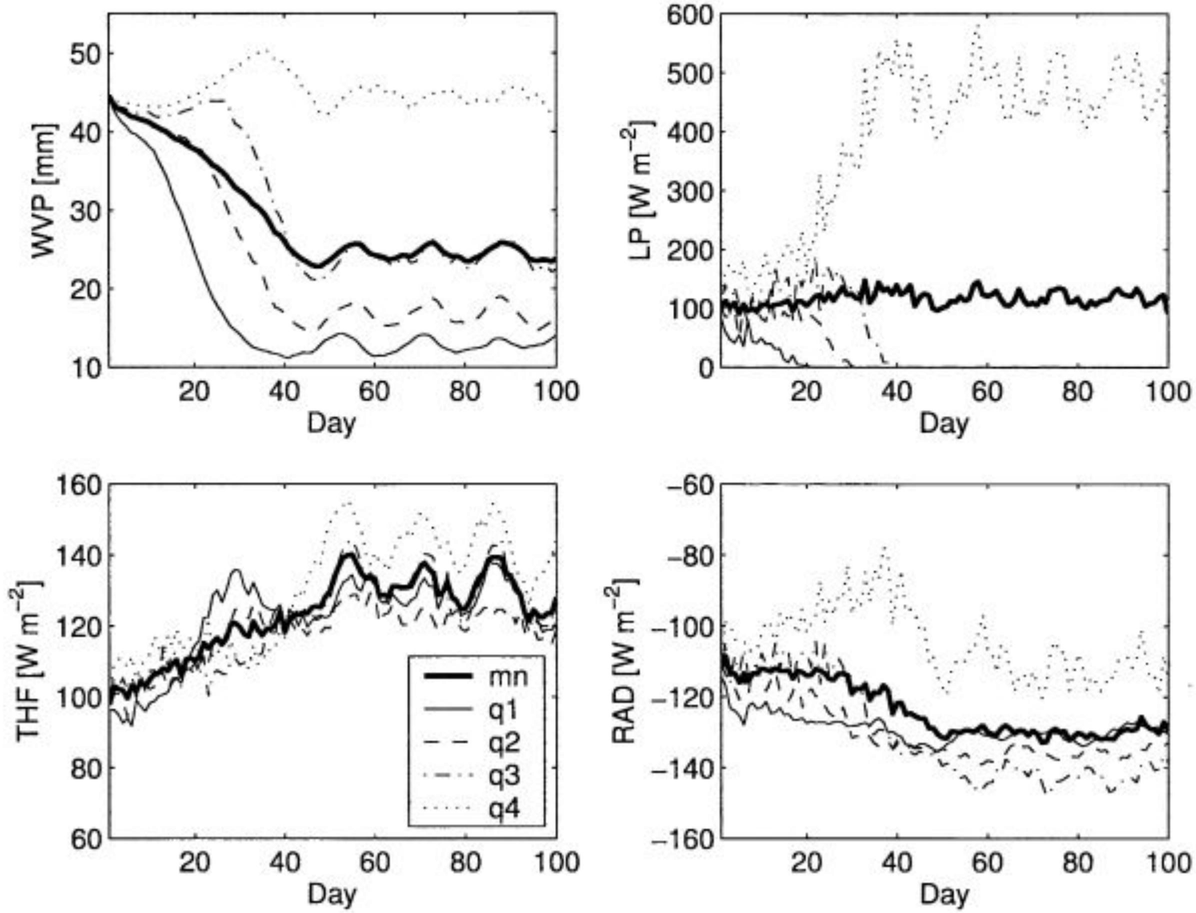
Simulated CSA



day 50

Bretherton et al., 2005

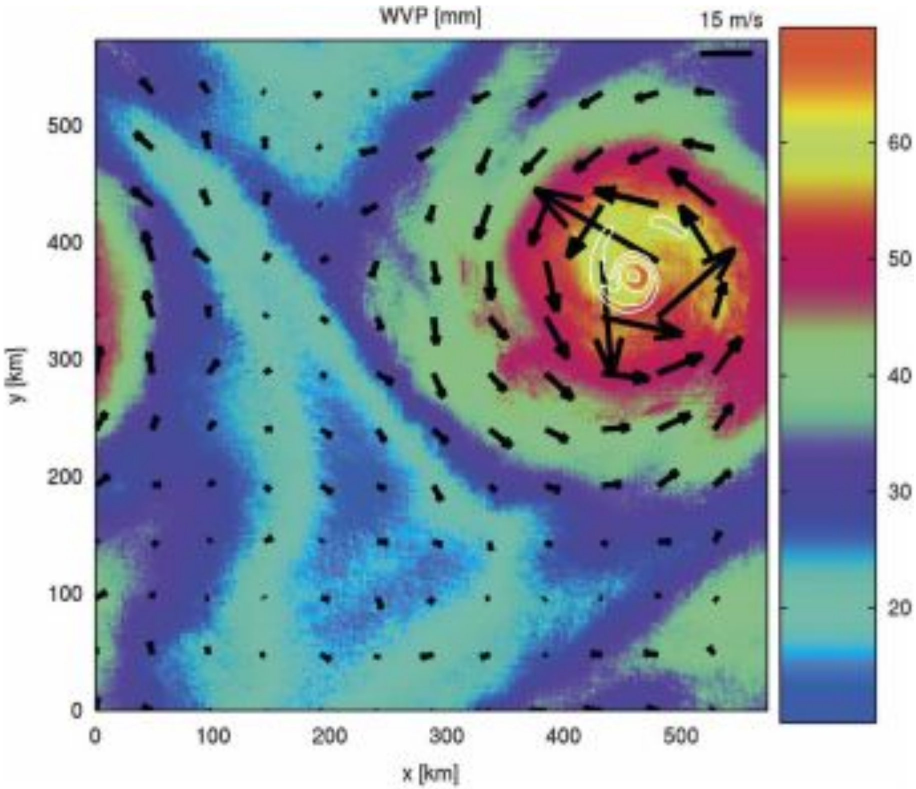
Simulated CSA



Bretherton et al., 2005

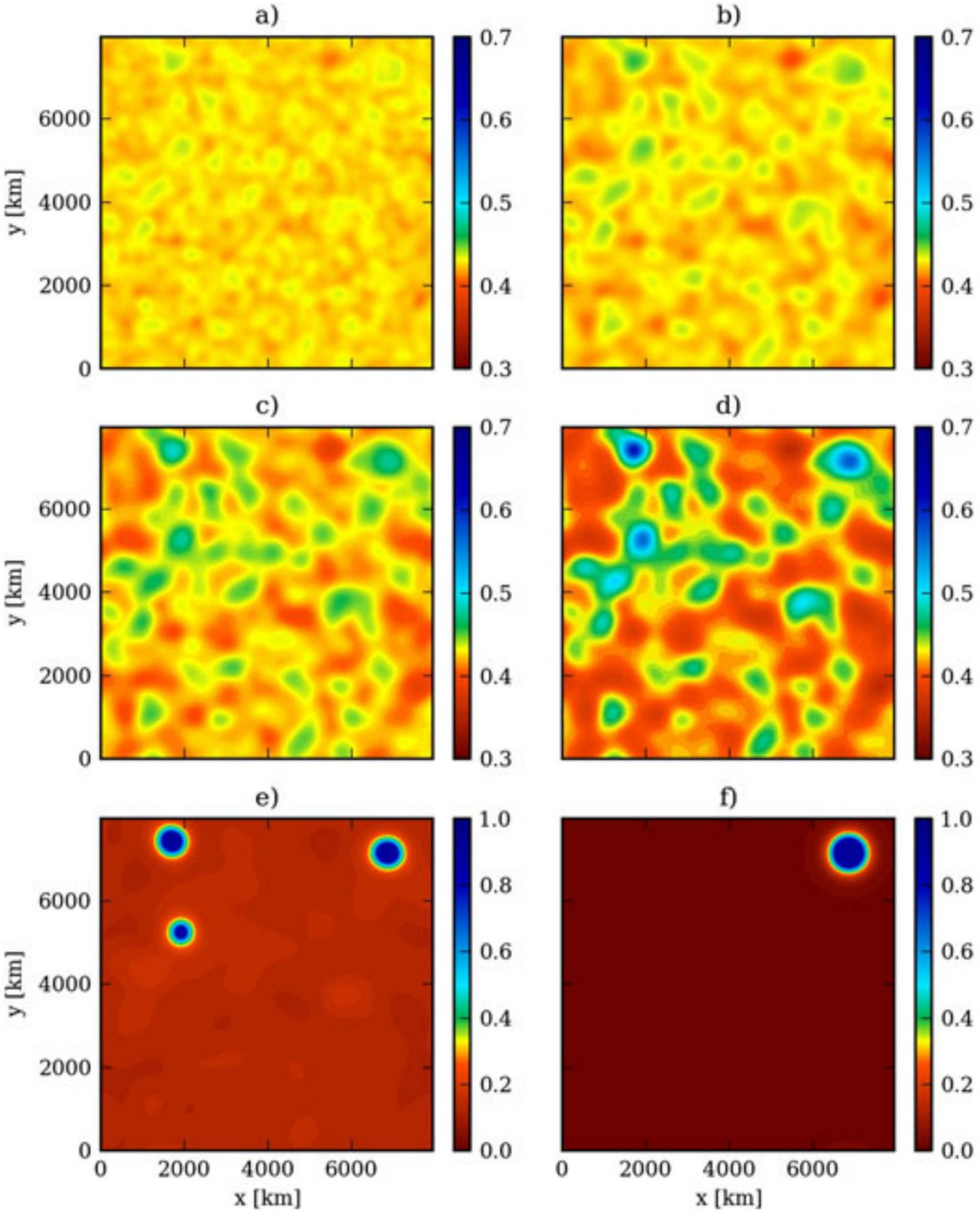
Introducing rotation (Coriolis)

Horizontal grid spacing: 3km, SST=301 K, 30 deg N (rotation)



Bretherton et al., 2005

Modeling CSA: coarsening



Moisture feedback

$$\frac{\partial I_v}{\partial t} = -\alpha I_v + a(t)\left(\frac{1}{\beta \frac{I_v}{I_v^*}} - 1\right)\left(e^{b\frac{I_v}{I_v^*}} - 1\right) + K\nabla^2 I_v$$

subsidence drying

rate of moistening

observed and modeled rainrate

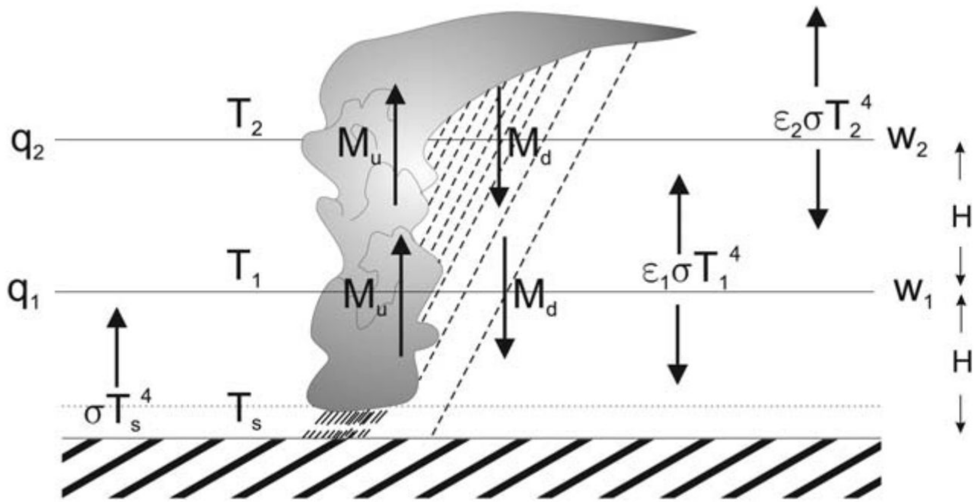
downgradient diffusive flux

global coupling:

$$a(t) = \frac{P_{av}}{\frac{1}{A}\int\left(e^{b\frac{I_v}{I_v^*}} - 1\right) dA}$$

Craig & Mack, 2013

Modeling CSA: radiation



$$\rho_1 \overline{\dot{Q}}_1 = -2\sigma\varepsilon_1 T_1^4 + \sigma\varepsilon_1\varepsilon_2 T_2^4 + \sigma\varepsilon_1 T_s^4,$$

$$\rho_2 \overline{\dot{Q}}_2 = -2\sigma\varepsilon_2 T_2^4 + \sigma\varepsilon_1\varepsilon_2 T_1^4 + \sigma\varepsilon_2 (1-\varepsilon_1) T_s^4$$

emissivity
monotonic increase
with q_i

Radiation feedback

$$L_v \begin{pmatrix} \dfrac{\partial q'_1}{\partial t} \\ \dfrac{\partial q'_2}{\partial t} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} \begin{pmatrix} q'_1 \\ q'_2 \end{pmatrix}$$

$$c_{11} \equiv \frac{\partial \dot{Q}_1}{\partial q_1}$$

$$c_{12} \equiv \frac{\partial \dot{Q}_1}{\partial q_2}$$

$$c_{21} \equiv \varepsilon_p \frac{S_2}{S_1} \frac{\partial \dot{Q}_1}{\partial q_1} + \frac{\partial \dot{Q}_2}{\partial q_1} (1-\varepsilon_p)$$

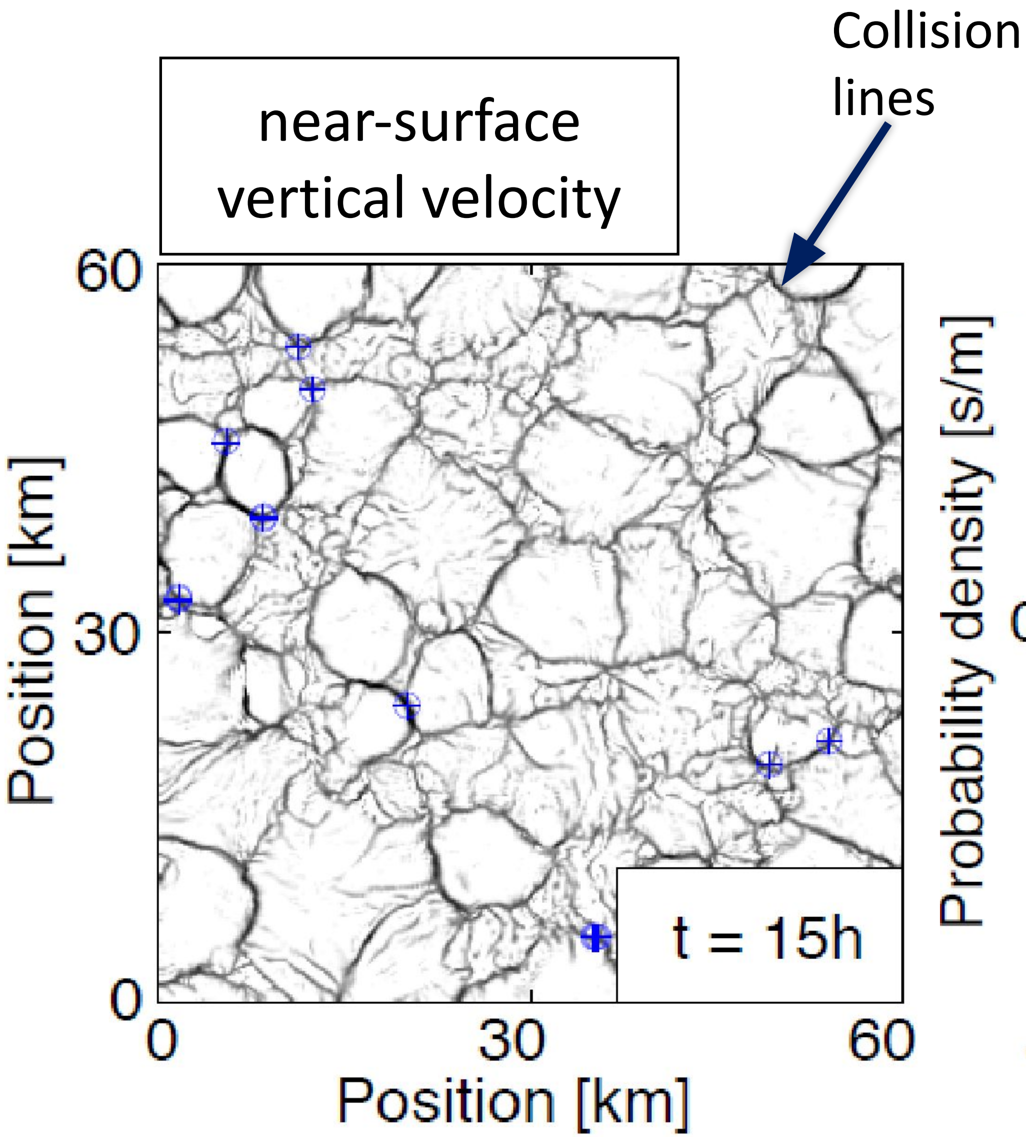
$$c_{22} \equiv \varepsilon_p \frac{S_2}{S_1} \frac{\partial \dot{Q}_1}{\partial q_2} + \frac{\partial \dot{Q}_2}{\partial q_2} (1-\varepsilon_p)$$

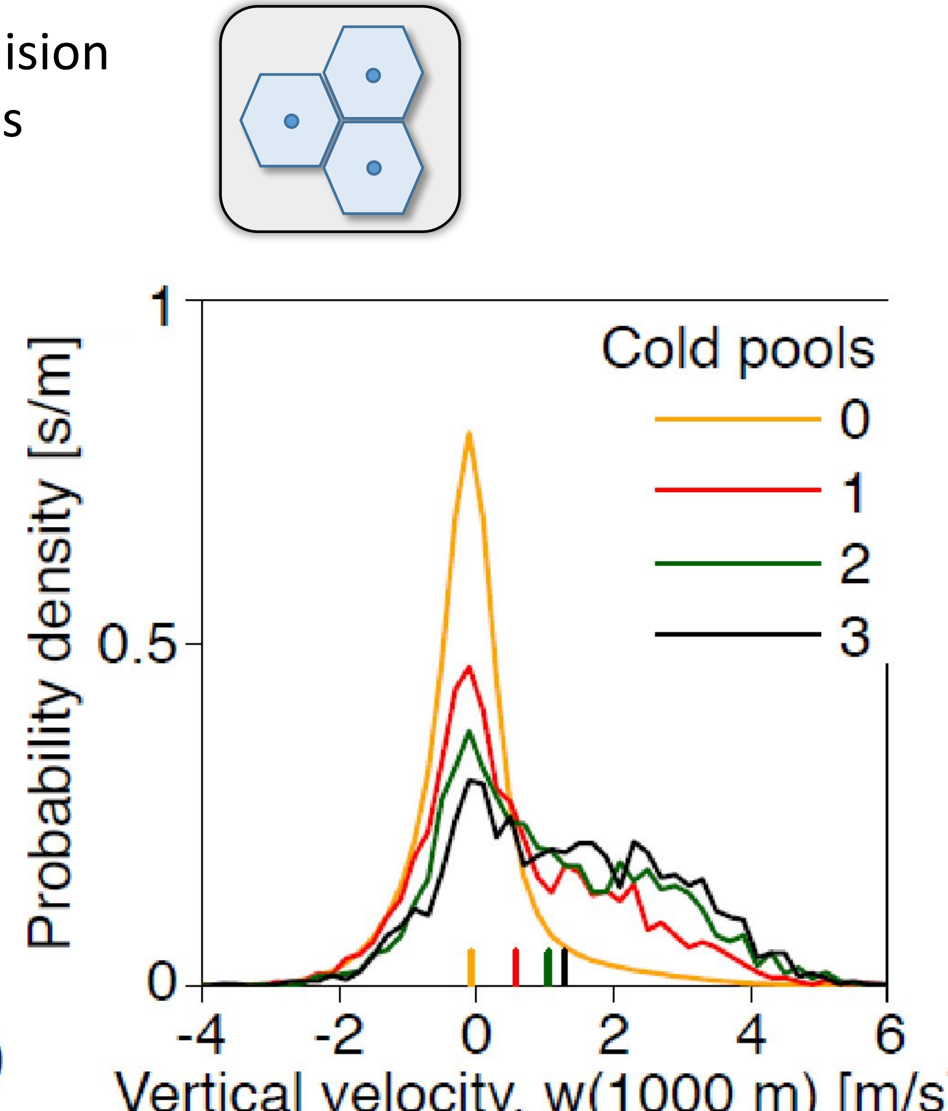
Emanuel et al., 2014

Proposed feedback mechanisms

- radiative feedback (Bretherton, 2005, Muller & Held, 2012, Emanuel et al., 2014)
- moisture feedback (Craig & Mack, 2013)
- interactions between rain cells (Haerter, 2019)

New events require collisions

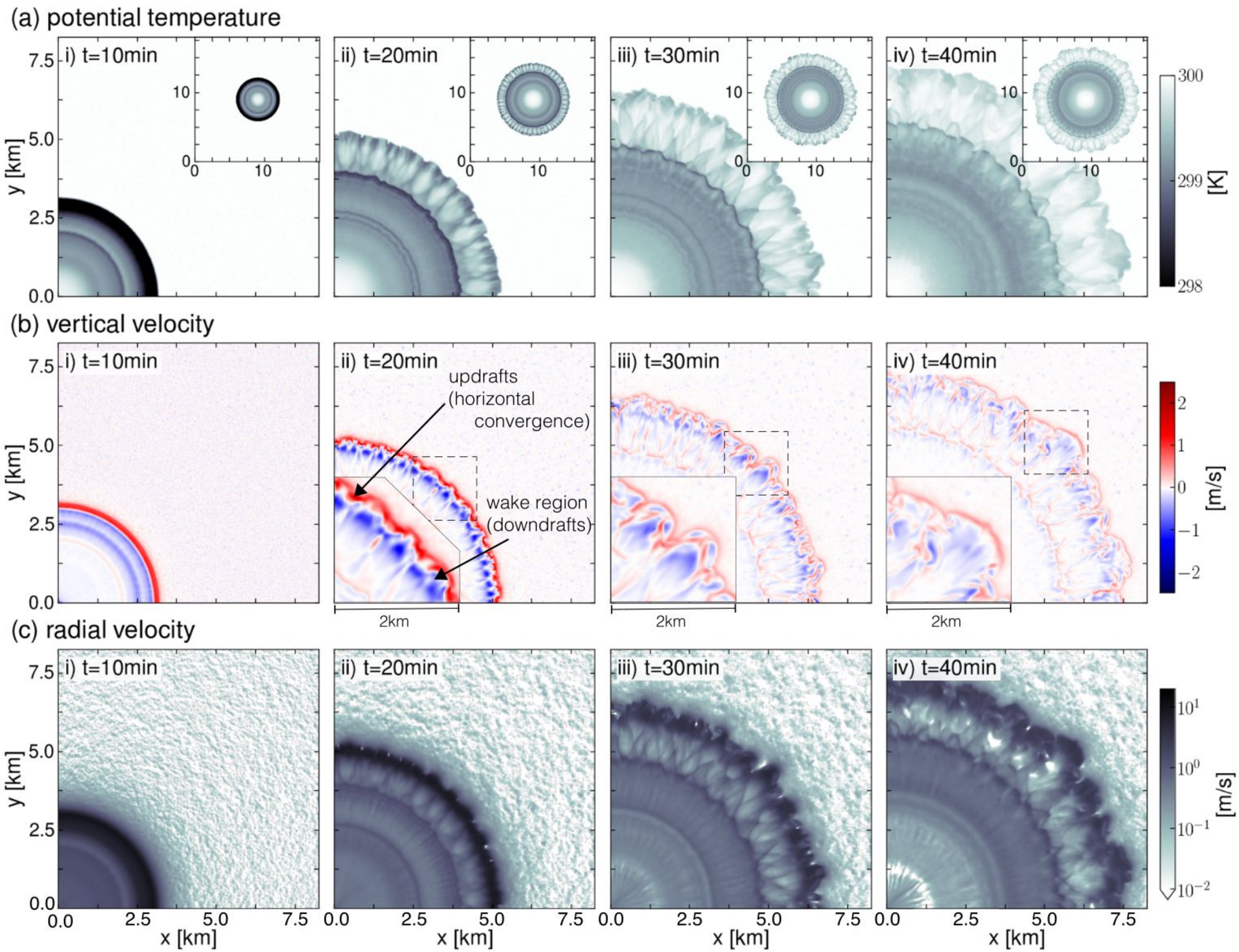




Haerter, Boeing, Henneberg, Nissen, *GRL, 2019*

Cold pools (simulation)

Meyer & Haerter, 2020



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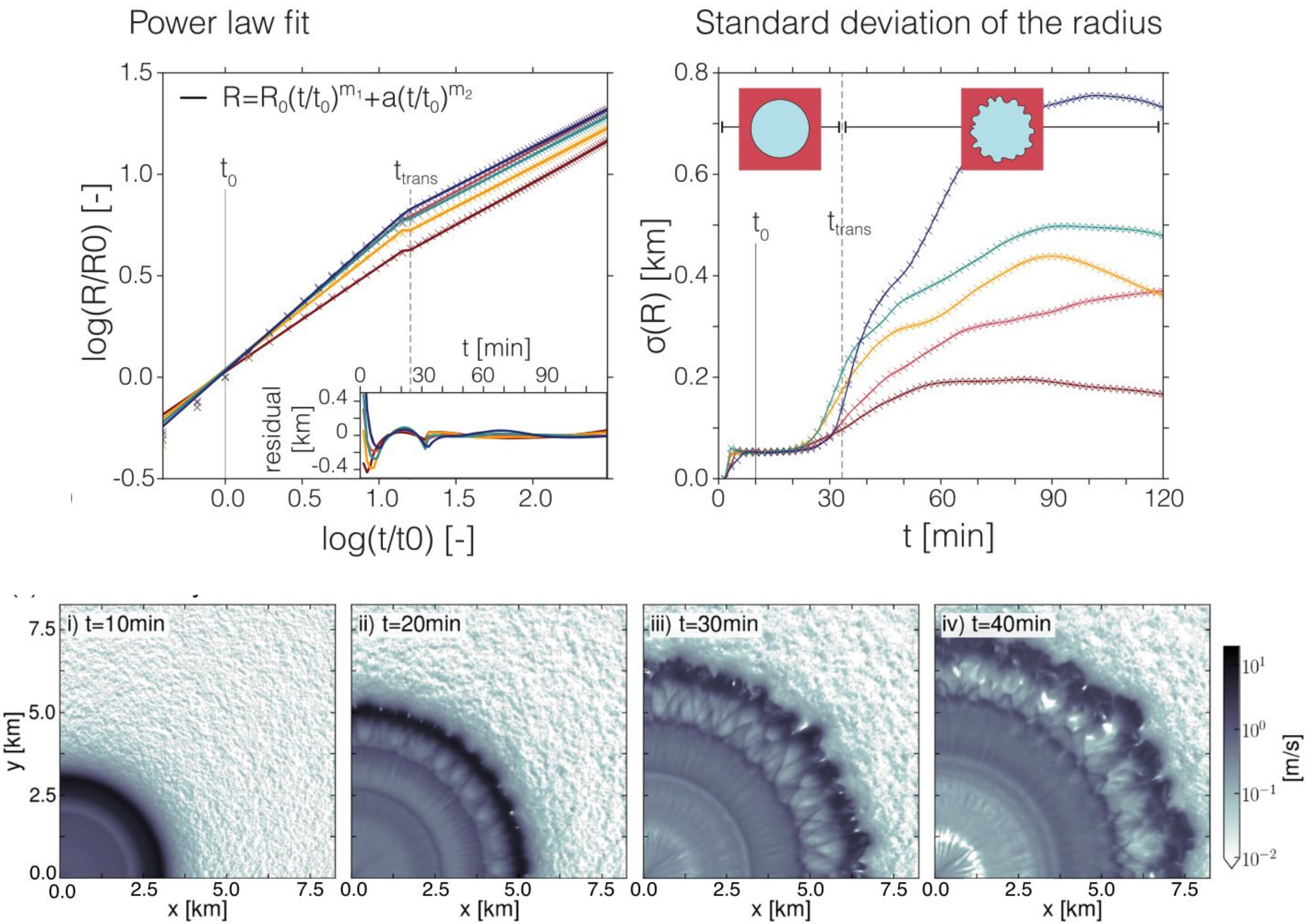
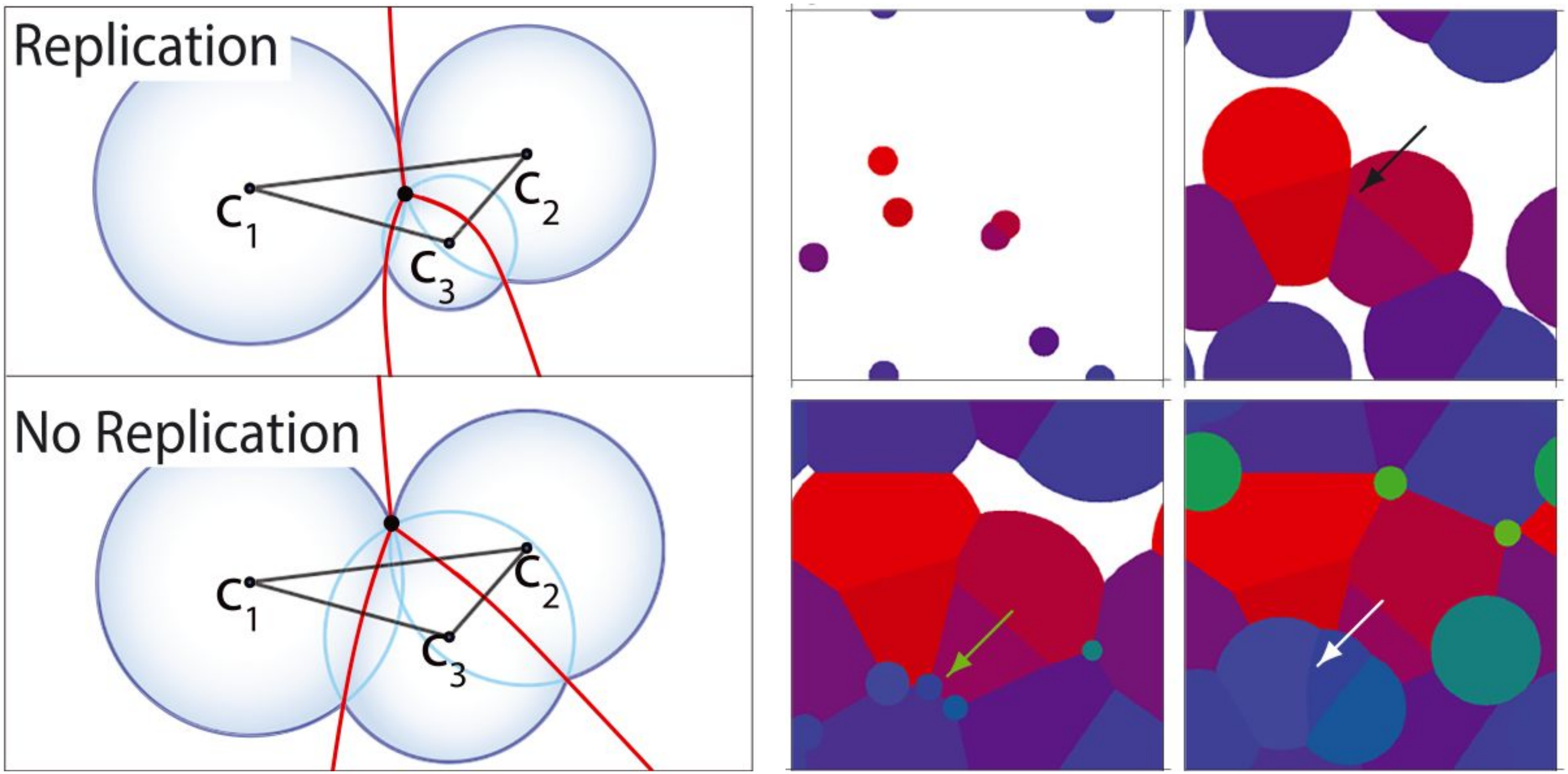


Simplified circle model

Haerter et al., 2019



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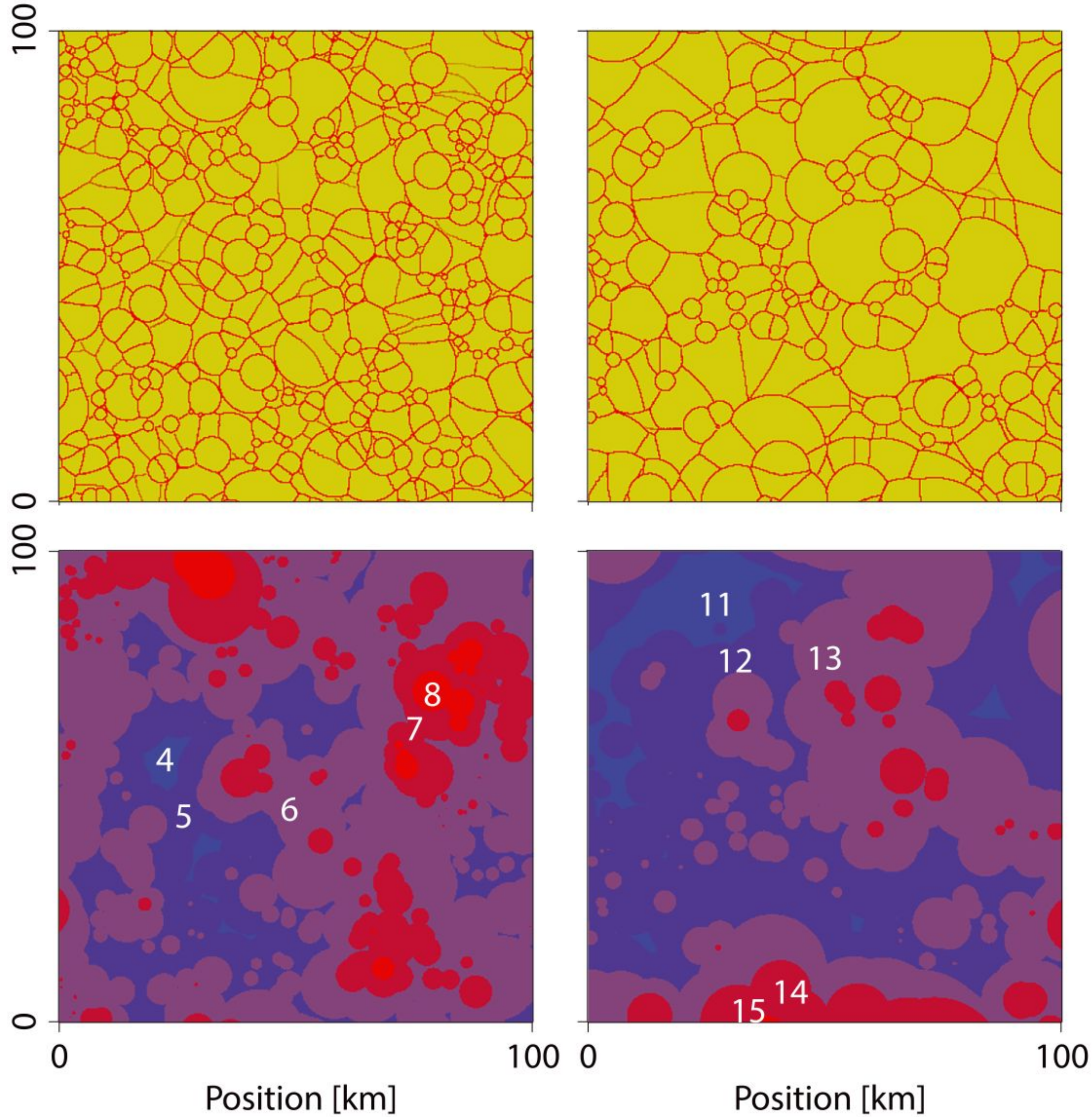


Modified Voronoi diagram

Generation view of cold pools

Simplified circle model

Haerter et al., 2019

scales change linearly with time

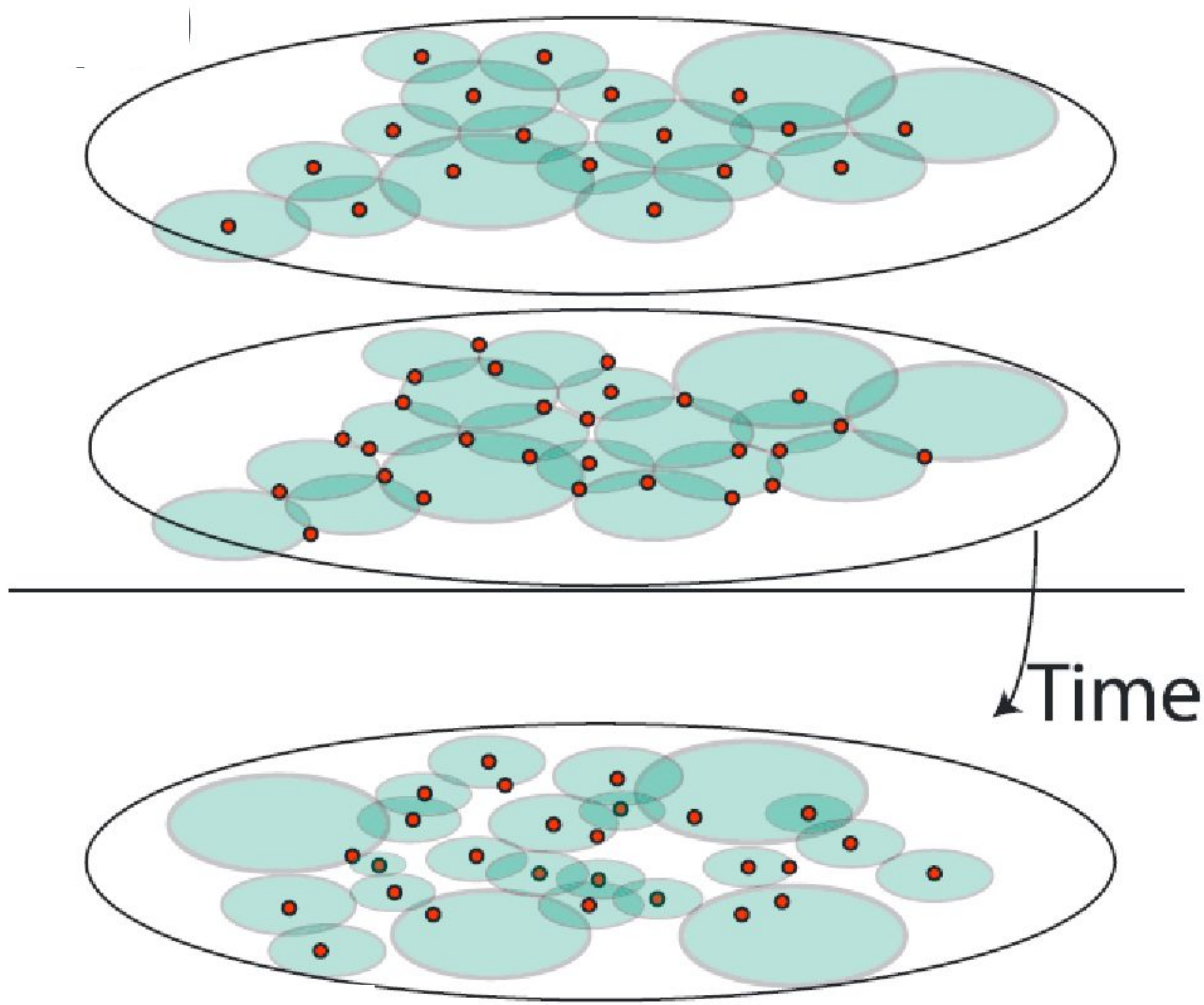
$$\Delta N = (r - 1)N(n)$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} \approx \frac{\Delta N}{\Delta t} = 2\, c_0 L^{-1}(r - 1)N^{3/2}$$

$$l(t) \equiv L\; N(t)^{-1/2} = l_0 + c_0\;(1 - r)t$$

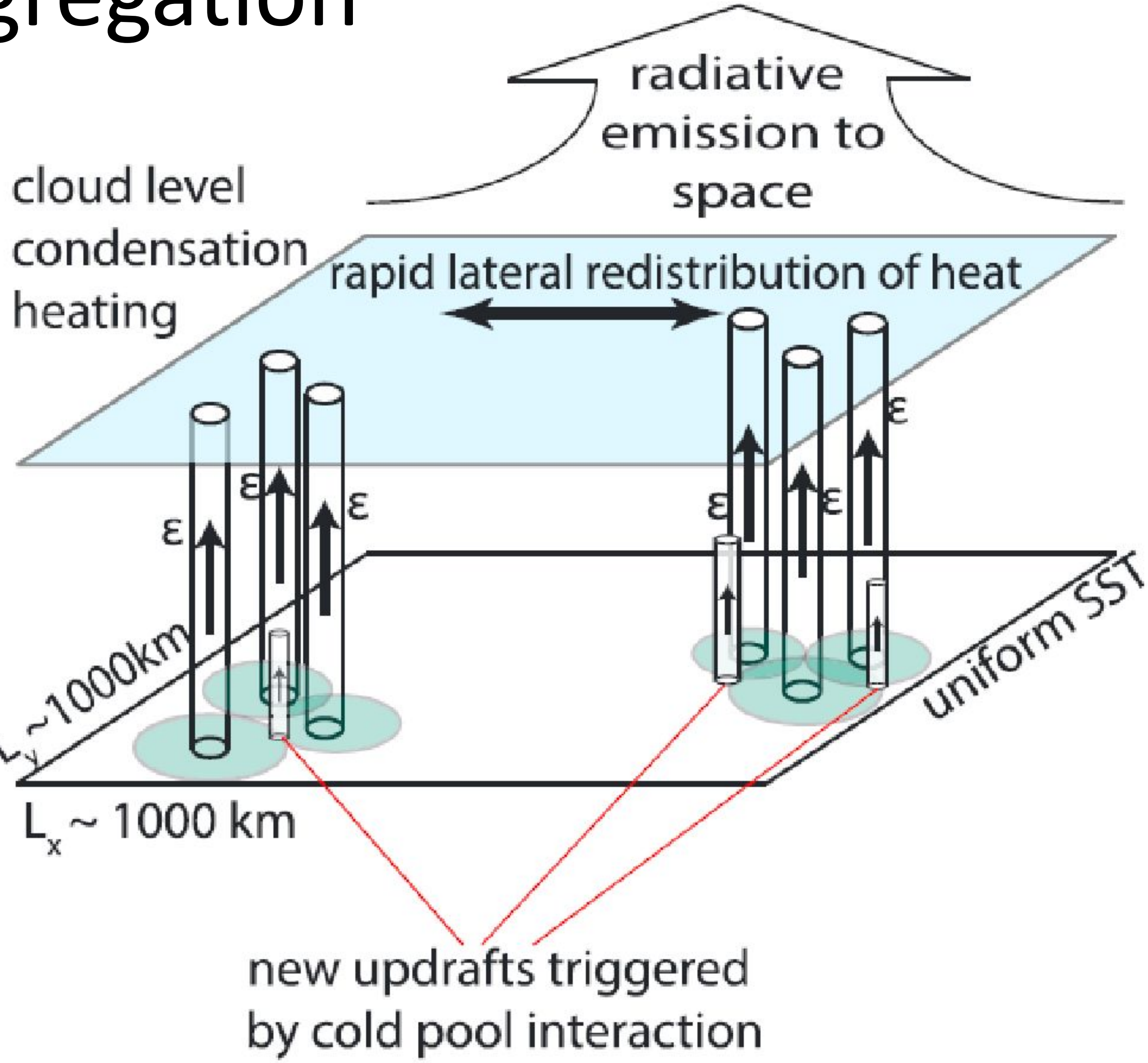
r depends on the details of the collision process

Replication of a cloud population



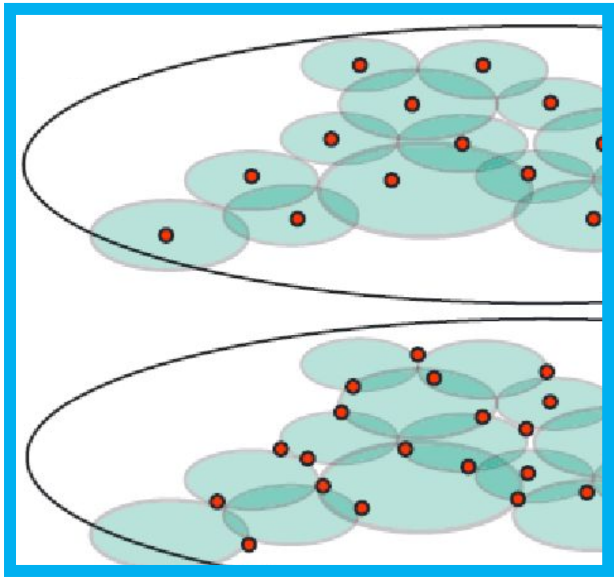
Haerter, *GRL, 2019*

A conceptual model for convective self-aggregation



Haerter, *GRL, 2019*

Cell number densities



Total number of cells

$$N \equiv \int_{\mathbf{r}} d\mathbf{r} \sum_i \delta(\mathbf{r} - \mathbf{c}_i)$$

cell centers

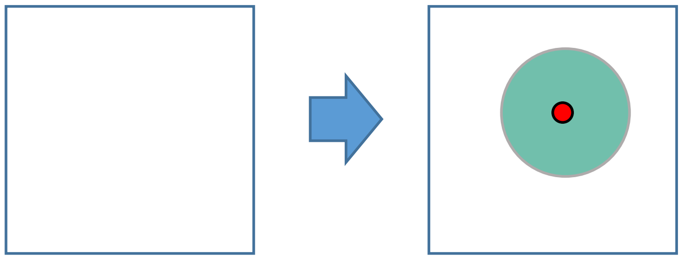
Local cold pool number density

$$\rho(\mathbf{r}) \equiv a\, a_{cp}^{-1} \int_{r=0}^{r_{max}} \int_{\phi=0}^{2\pi} dr d\phi \sum_i \delta(\mathbf{r} + \mathbf{r}' - \mathbf{c}_i)$$

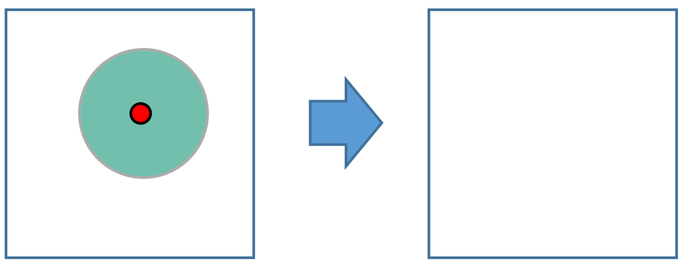
Haerter, *GRL, 2019*

Three model ingredients

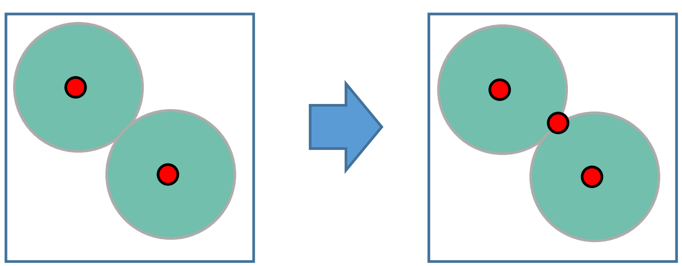
1. **s**pontaneous cell **g**eneration $$f_{sg}(1-\overline{\rho})(1-\rho(\mathbf{r}))$$



2. cell **d**ecay $$-f_d\rho(\mathbf{r})$$



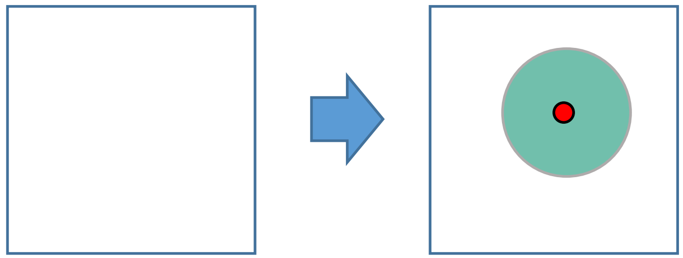
3. cell interaction $$p_0(1-\overline{\rho})\rho(\mathbf{r})^m(1-\rho(\mathbf{r}))$$



Three model ingredients

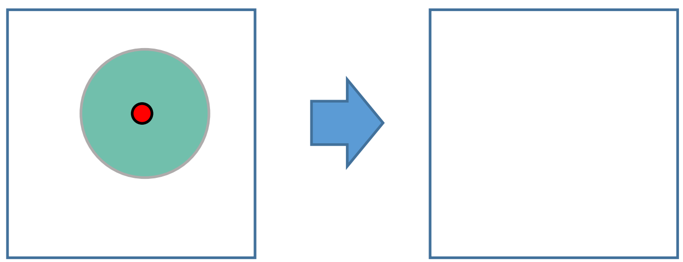
1. spontaneous cell **g**eneration

$$f_{sg}(1-\bar{\rho})(1-\rho(\mathbf{r}))$$



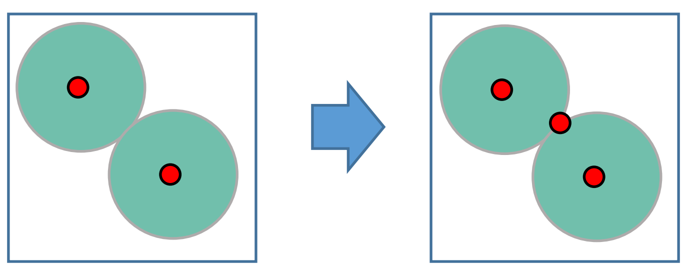
2. cell **d**ecay

$$-f_d\rho(\mathbf{r})$$



3. cell interaction

$$p_0\underbrace{(1-\bar{\rho})}_{\text{global energy constraint (RCE)}}\rho(\mathbf{r})^m\underbrace{(1-\rho(\mathbf{r}))}_{\text{local space limitation}}$$



Three model ingredients

$$\frac{d}{dt}\rho(\mathbf{r},t) = \underbrace{\overbrace{p_0(1-\overline{\rho})\rho(\mathbf{r})^m(1-\rho(\mathbf{r}))}^{\text{cell interaction}} + \overbrace{f_{sg}(1-\overline{\rho})(1-\rho(\mathbf{r}))}^{\text{spontaneous cell generation}} - \overbrace{f_d\,\rho(\mathbf{r})}^{\text{cell decay}}}$$

$$\frac{d}{dt}\rho(\mathbf{r},t) = F_{q,p_0,f_{sg}}(\rho) \equiv q\,(1-\rho)(p_0\,\rho^m + f_{sg}) - \rho$$

$$q = 1 - \overline{\rho}$$

m=1 Diffusion-like dynamics

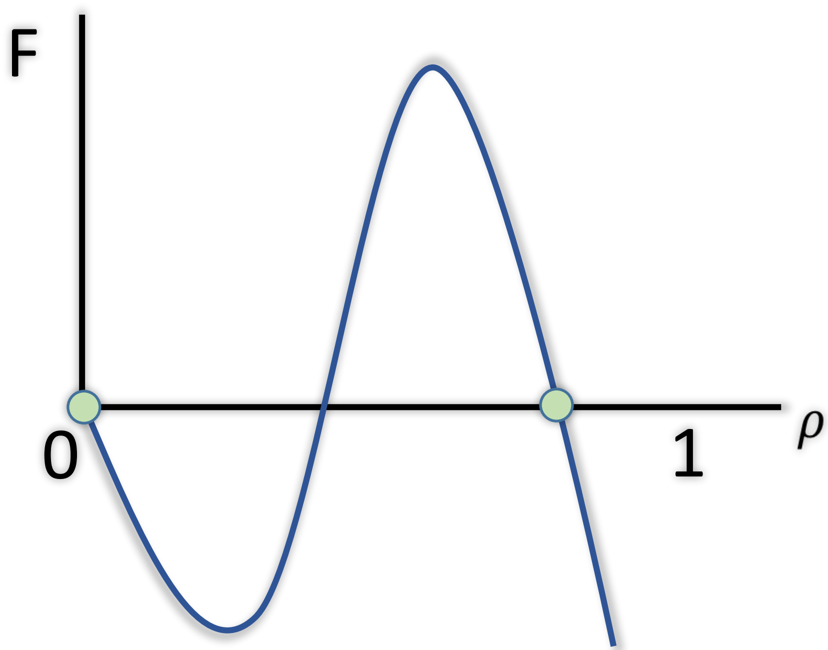
m=2 Spatial interaction

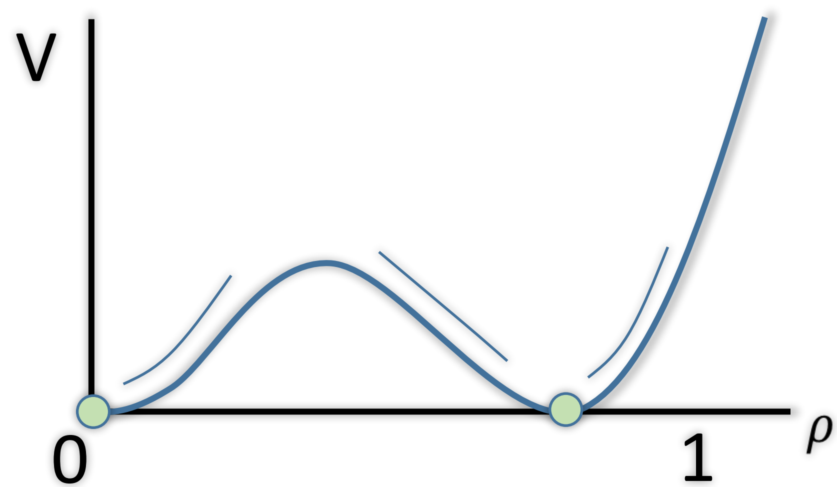
Simpler case:
ignoring spontaneous initiation

$$F_{q,p_0,f_{sg}}(\rho) \quad \equiv \quad q\,(1-\rho)(p_0\,\rho^m + \cancelto{0}{f_{sg}}) - \rho \qquad m\text{=}2$$

$$F_{q,p_0}(\rho) \equiv -\rho\, q\, p_0\,(\rho^2 - \rho + 1/(q\,p_0))$$

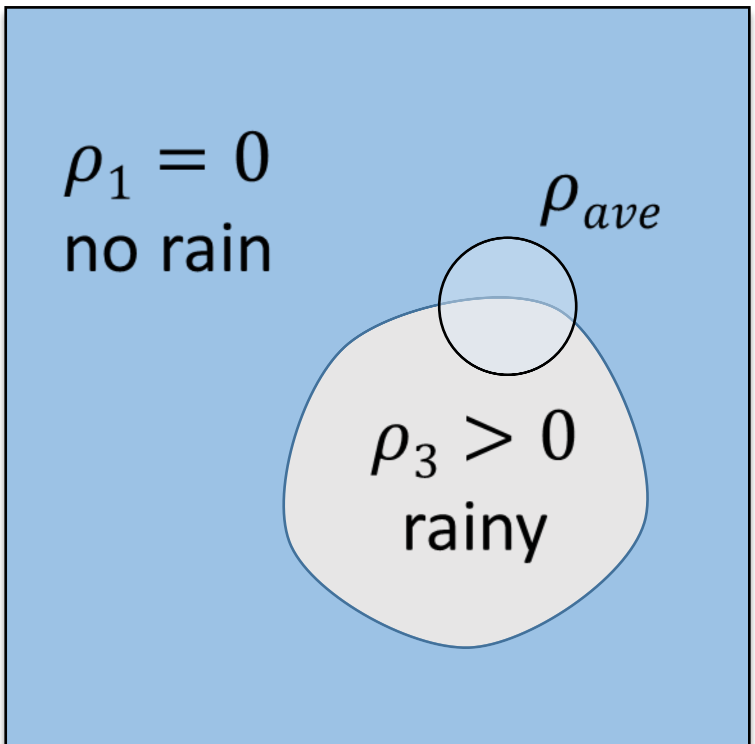
F(ρ) = - d V/d
ρ





Looking for a segregated steady state

$$F_{q,p_0}(\rho) \equiv -\rho\, q\, p_0\, (\rho^2 - \rho + 1/(q\, p_0))$$



$$\boxed{\rho_{ave} \equiv (\rho_1 + \rho_3)/2 = \rho_3/2}$$

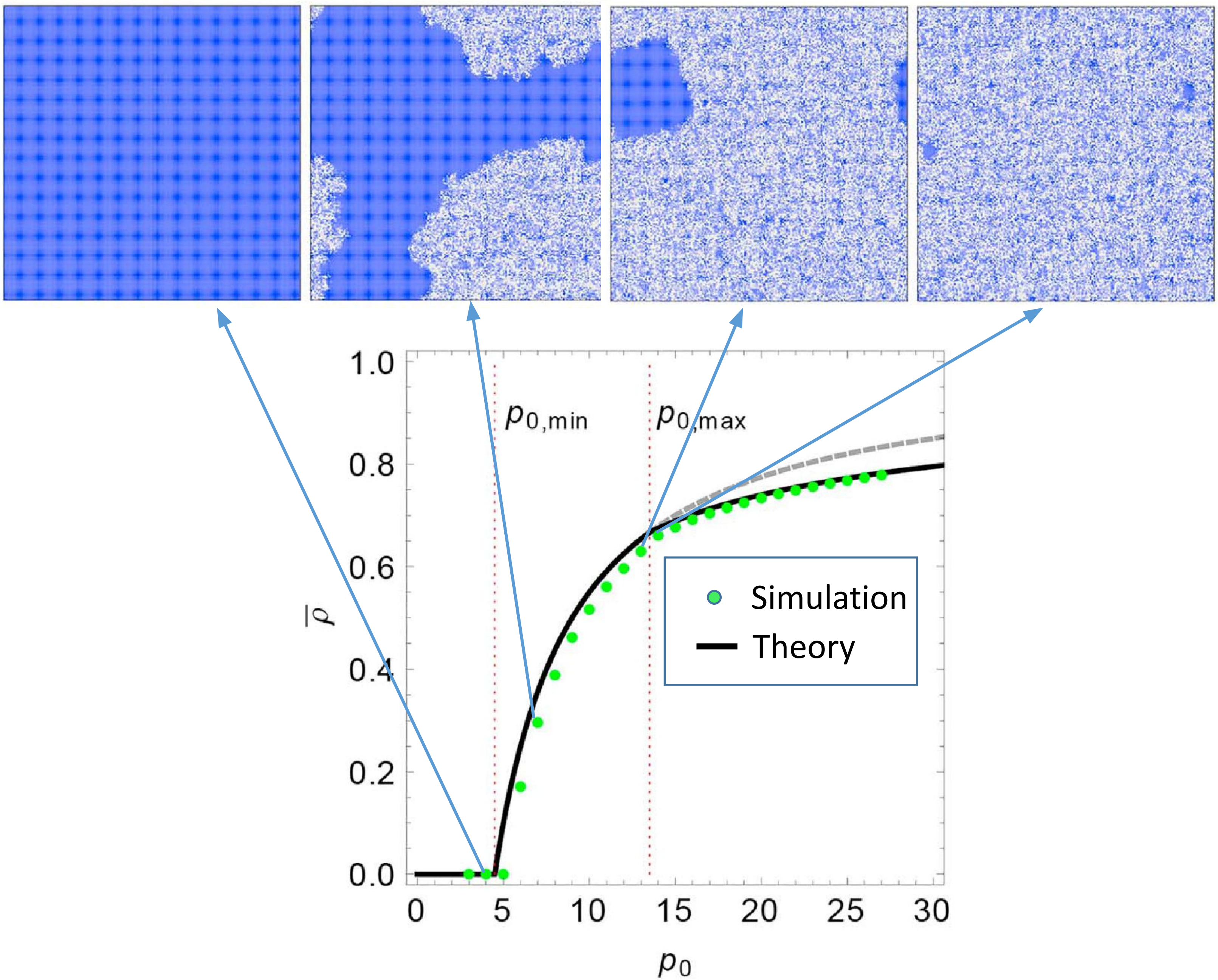
$$\rho_{2,3} = \frac{1}{2} \mp \left(\frac{1}{4} - \frac{1}{p_0 q}\right)^{\frac{1}{2}}$$

$$q = 1 - \bar{\rho} = \frac{9}{2} p_0^{-1}$$

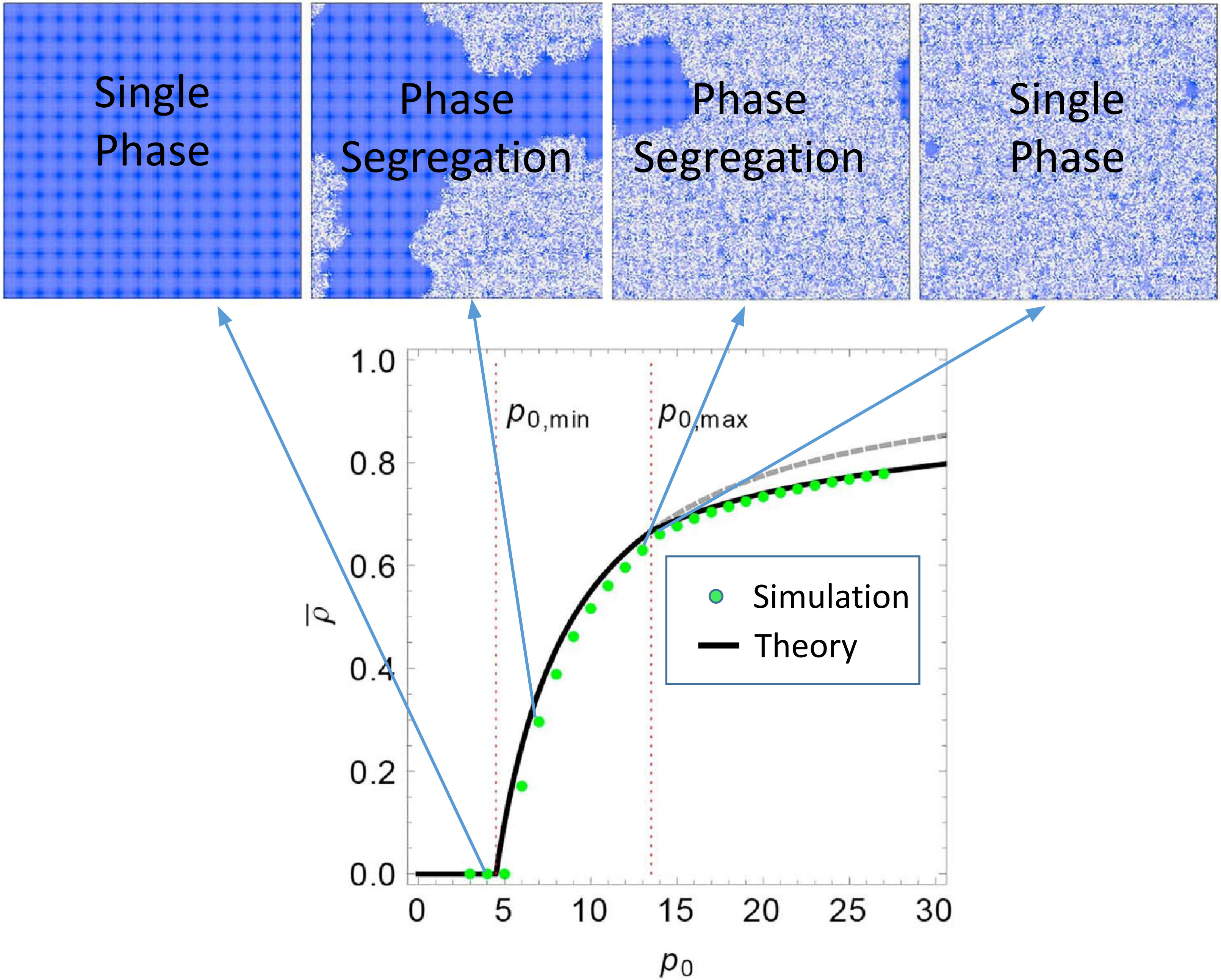
$$\rho_1 < 1 - q < \rho_3$$

$$\frac{9}{2} \equiv p_{0,min} < p_0 < p_{0,max} \equiv \frac{27}{2}$$

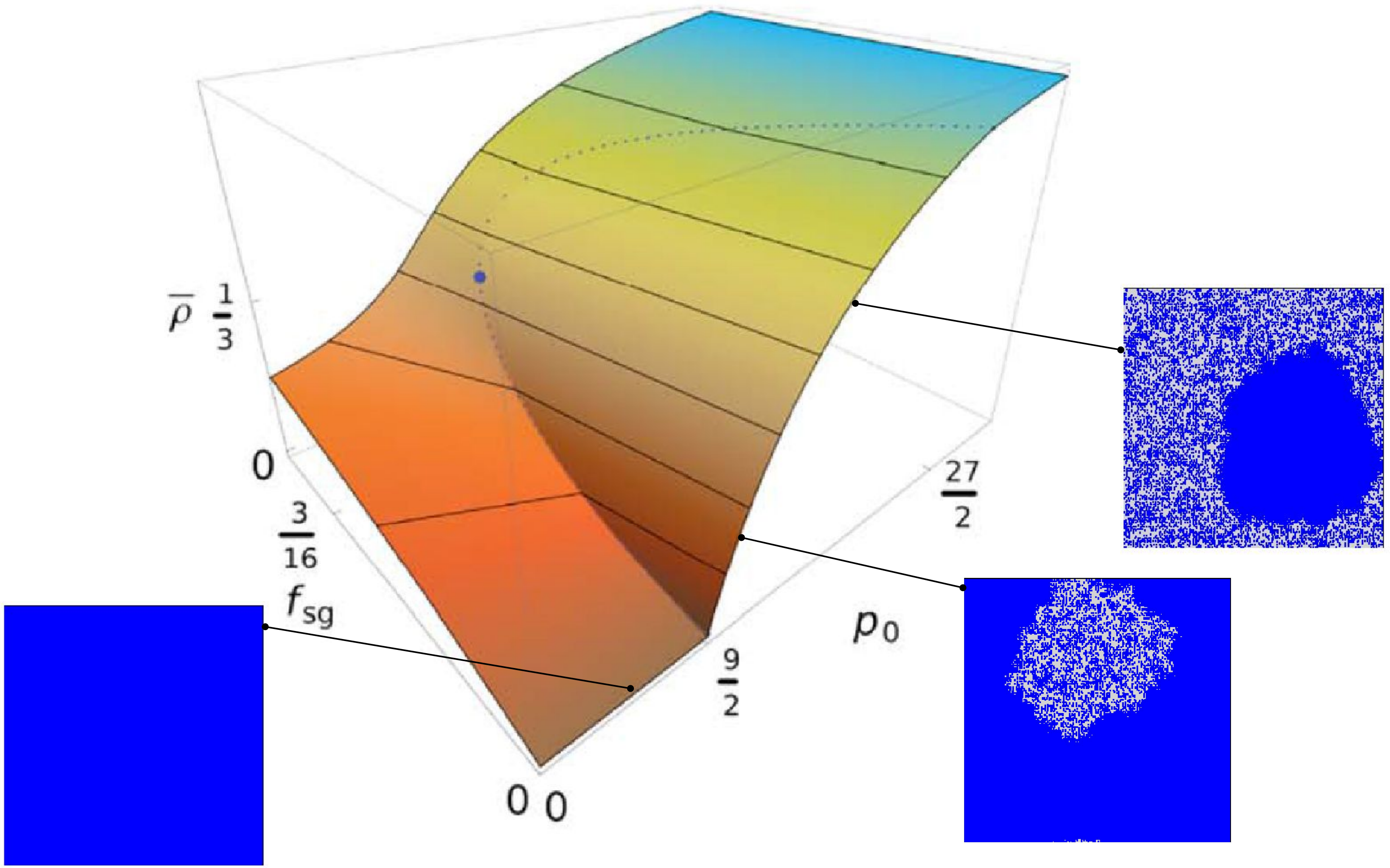
Organization as function of interaction



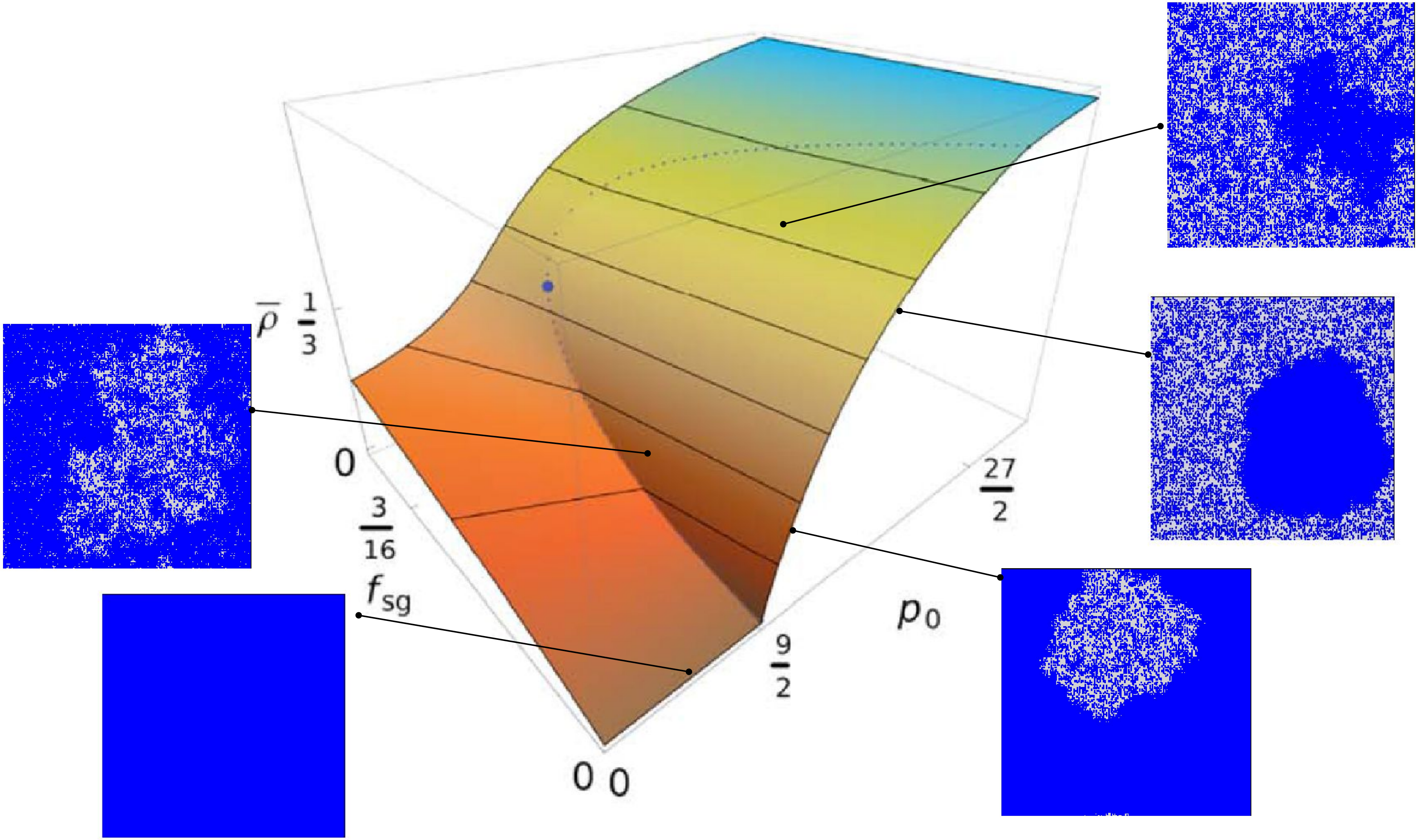
Organization as function of interaction



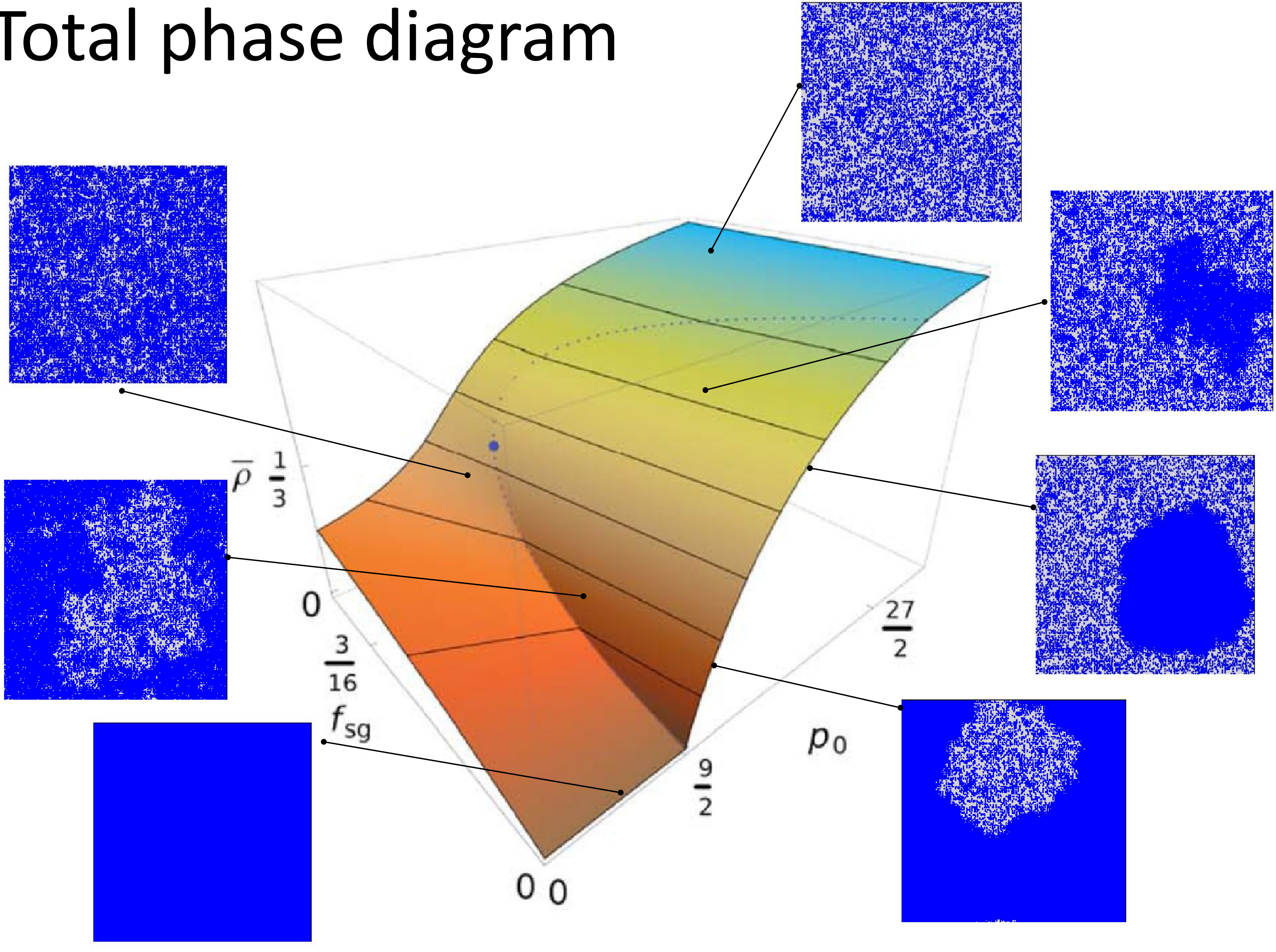
Total phase diagram



Total phase diagram



Total phase diagram



Summary

- Convective self-aggregation from cold pool interaction and a global energy constraint
- Conceptual model for a phase transition between a uniform and an aggregated state – dependent on cold pool interaction
- Bifurcation analogous to a continuous phase transition

References

Haerter, J. O. (2019) *Geophysical Research Letters*, *46*, 4017–4028.

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- Convective self-aggregation from cold pool interaction and a global energy constraint
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Thank you for your attention!

References

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