Jacobs University
Fall 2017

September 11, 2017

Advanced Calculus (Element of Analysis)

Homework 1

Due on September 18, 2017

Note: The work has to be handed in individually on the due date. Directly copying homework sheets is a violation of Academic Integrity! However, discussions about the problems are encouraged. You need to show your work, i.e., just writing down the solution does not give full points. Write down the calculation steps, i.e., show how you came up with the solution.

Problem 1 [5 points]

Consider the equation of degree four with symmetric coefficients, i.e.,

$$x^4 + ax^3 + bx^2 + ax + 1 = 0,$$

for some parameters $a, b \in \mathbb{R}$. Under which condition on a and b does it admit a decomposition

$$(x^2 + rx + 1)(x^2 + sx + 1)$$

for some parameters $r, s \in \mathbb{R}$? Using this decomposition, find the roots of the equation

$$x^4 + 5x^3 + 8x^2 + 5x + 1 = 0.$$

Problem 2 [5 points]

Prove by induction in n that

$$\sum_{k=0}^{n} \binom{m-1+k}{k} = \binom{m+n}{n} \quad \text{given any } m \geq 1,$$

and that

$$\sum_{k=0}^{n-1} \binom{m+k}{m} = \binom{m+n}{m+1} \quad \text{given any } m \geq 0.$$

Problem 3 [5 points]

Prove either by induction or by any other means that

$$\sum_{k=0}^{n} k = \frac{n(n+1)}{2} \qquad \text{and} \qquad \sum_{k=0}^{n-1} x^k = \frac{1-x^n}{1-x} \quad \text{for all } x \neq 1.$$

Problem 4 [5 points]

Prove the inequality of arithmetic and geometric means

$$xy \leq \left(\frac{x+y}{2}\right)^2$$

and Bernoulli's inequality

$$(1+x)^n \geq 1+nx \quad \text{for } x \geq -1 \text{ and all } n \in \mathbb{N}.$$