Jacobs University October 02, 2017
Fall 2017

Advanced Calculus (Element of Analysis)

Homework 4

Due on October 09, 2017

Problem 1 [4 points]
Let $z_1 = 1+2i$ and $z_2 = 3-2i$. Calculate $z_1 z_2$, $\frac{z_1}{z_2}$, and give the result in polar representation (pay attention to what the right angle is) and state the real and imaginary part. Draw z_1, z_2, $z_1 z_2$ and $\frac{z_1}{z_2}$ in the complex plane.

Problem 2 [3 points]
Calculate all solutions to the following equations:

(a)
$$z^2 = i,$$

(b)
$$z^3 = -1,$$

(c)
$$z^i = i.$$

Problem 3 [3 points]
Find the roots of
$$x^2 + 4x + 13 = 0.$$

Problem 4 [4 points]
In the first lecture, we learned a recipe to find the zeros of polynomials of degree 3 due to Vieta: For an equation of the type

$$x^3 + px + q = 0,$$

set $x = z - \frac{p}{3z}$, and then solve the resulting equation which is quadratic in z^3. Use this recipe to find the roots of
$$x^3 - 3\sqrt[3]{4}\, x - 2 = 0.$$

Problem 5 [2 points]
Sum up the expression

$$D_n(x) = \sum_{k=-n}^{n} e^{ikx}.$$

What is the behavior of $D_n(x)$ for very small x as $n \to \infty$?

Problem 6 [4 points]

(a) Express $\sin(x)$ and $\cos(x)$ in terms of $e^{\pm ix}$ by using the Euler formula.

(b) Find the relation between $\sin(ix)$, $\cos(ix)$ and $\sinh(x)$ and $\cosh(x)$.

(c) Using (a), prove the identity

$$\sin^2\left(\frac{x}{2}\right) = \frac{1-\cos(x)}{2}.$$