

Advanced Calculus

Homework 4

Due on October 8, 2018

Problem 1 [4 points]

Let $z_1 = 2 + i$ and $z_2 = 4 - 3i$. Calculate $z_1 z_2$, $\frac{z_1}{z_2}$, and give the results in polar representation (pay attention to what the right angle is) and state the real and imaginary part. Draw z_1 , z_2 , $z_1 z_2$ and $\frac{z_1}{z_2}$ in the complex plane.

Problem 2 [2 points]

Find the roots of

$$x^2 + 4x + 13 = 0.$$

Problem 3 [3 points]

Calculate all solutions to the following equations:

(a)

$$z^2 = i,$$

(b)

$$z^3 = -1,$$

(c)

$$z^i = i.$$

Problem 4 [4 points]

In the first lecture, we learned a recipe to find the zeros of polynomials of degree 3 due to Vieta: For an equation of the type

$$x^3 + px + q = 0,$$

set $x = z - \frac{p}{3z}$, and then solve the resulting equation which is quadratic in z^3 . Use this recipe to find the roots of

$$x^3 - 3\sqrt[3]{4}x - 2 = 0.$$

Problem 5 [3 points]

- (a) Express $\sin(x)$ and $\cos(x)$ in terms of $e^{\pm ix}$ by using the Euler formula.
- (b) Find the relation between $\sin(ix)$, $\cos(ix)$ and $\sinh(x)$ and $\cosh(x)$.
- (c) Using (a), prove the identity

$$\sin^2\left(\frac{x}{2}\right) = \frac{1 - \cos(x)}{2}.$$

Problem 6 [4 points]

Sum up the expression

$$D_n(x) = \sum_{k=-n}^n e^{ikx}.$$

What is the behavior of $D_n(x)$ for very small x as $n \rightarrow \infty$?