Advanced Calculus

Homework 5

Due on October 15, 2018

Problem 1 [7 points]

Calculate the derivatives of the following functions:

$$f_1(x) = x^3 + 5x^2 + 3x + 5, \quad f_2(x) = e^x \sin(x), \quad f_3(x) = \frac{x^3}{\sqrt{1-x}}, \quad f_4(x) = \frac{\ln(x)}{x^3},$$

$$f_5(x) = \ln(a^x + a^{-x}) \ \text{for any } a > 0, \quad f_6(x) = \sum_{k=0}^{\infty} k^3 \frac{x^k}{k!}$$

$$f_7(x) = \arccos(x),$$

where arccos is the inverse of cos; for f_7, express the result without using trigonometric functions.

Problem 2 [2 points]

Prove that the function $f(x) = e^{|x|}$ is not differentiable at $x = 0$.

Problem 3 [2 points]

In class we discussed the product rule $(fg)' = f'g + fg'$. Generalize this rule for the n-th derivative $(fg)^{(n)}$.

Problem 4 [2 points]

In class, we discussed a theorem by Lagrange: for continuous functions $f : [a, b] \to \mathbb{R}$, which are differentiable on (a, b), there is an $m \in (a, b)$ such that

$$f(b) - f(a) = f'(m)(b - a).$$

Now, consider the function
$$f(x) = 2x^2 + 3x + 1,$$

and $a = 0$, $b = 1$. Find an m such that the equation above holds.

Problem 5 [2 points]

An ellipse is described by the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

for two fixed numbers a and b. Calculate the derivative $\frac{dy}{dx}$ by implicit differentiation.

Problem 6 [5 points]

Consider the curve parametrized by φ where $x = a\cos(3\varphi)$ and $y = a\cos(\varphi)$ for some given $a \in \mathbb{R}$.

(a) Calculate $\frac{dy}{dx}$ using the given parametrization.

(b) Show that the curve satisfies $4y^3 = a^2(x + 3y)$.

(c) Calculate $\frac{dy}{dx}$ by implicit differentiation of the equation from (b). Does the result coincide with (a)?