Jacobs University
Fall 2018

September 17, 2018

# Linear Algebra

## Homework 2

Due on September 24, 2018

**Problem 1 [5 points]**
Let $V$ and $W$ be vector spaces over a field $F$. Prove that $\mathcal{L}(V, W)$, the space of all linear maps $V \to W$, is a vector space (with the usual addition and scalar multiplication of maps).

**Problem 2 [5 points]**
Let $\{v_1, v_2, \ldots\}$ be a basis of the infinite dimensional vector space $V$. Show that the elements $\{v_1^*, v_2^*, \ldots\}$ of the dual space $V^*$, defined by $v_i^*(v_j) = \delta_{ij}$, are linearly independent but not necessarily a basis.

**Problem 3 [4 points]**
Let $V$ be a one-dimensional vector space over a field $F$, and let $f \in \mathcal{L}(V)$. Prove that then there exists a $c \in F$ such that $f(v) = cv$ for all $v \in V$.

**Problem 4 [2 points]**
Give an example of a function $f : \mathbb{R}^2 \to \mathbb{R}$ such that

$$f(cv) = cf(v)$$

for all $c \in \mathbb{R}, v \in \mathbb{R}^2$, but which is not linear.

**Problem 5 [4 points]**
Let $V$ and $W$ be finite dimensional vector spaces, let $U \subset V$ be a subspace and let $f \in \mathcal{L}(U, W)$. Prove that there exists a map $g \in \mathcal{L}(V, W)$ which coincides with $f$ on the subspace $U$, i.e., which is such that $f(u) = g(u)$ for all $u \in U$.