1.5 Spot Rates

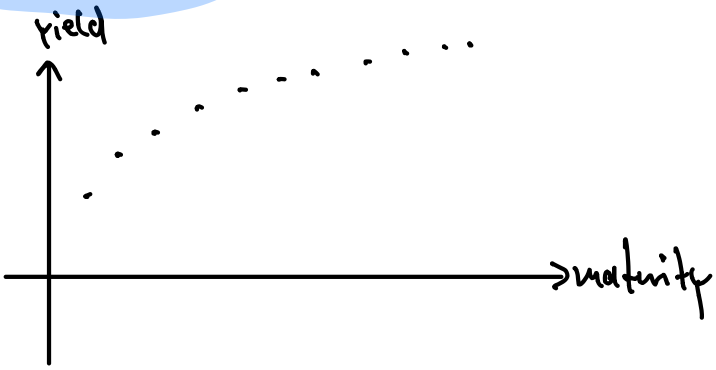
Session 5
Sep. 20, 2018

yield should be different depending on maturity date

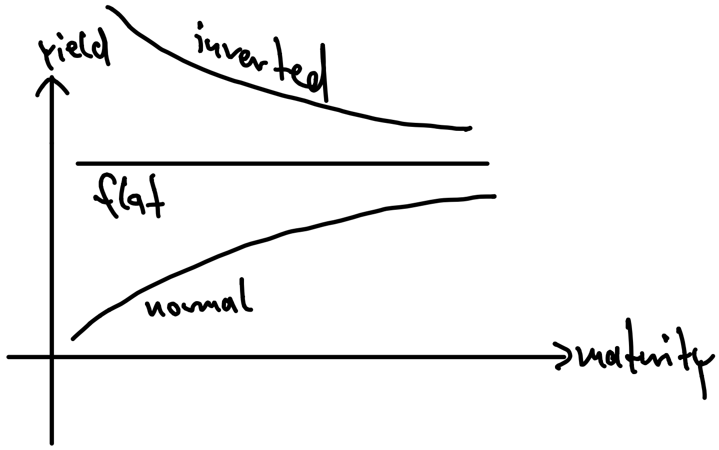
(usually: longer commitment $\Rightarrow$ more interest)

$\Rightarrow$ called "term structure"

yield curve:



types of curves:



spot rate $S(i)$:= yield to maturity of i-period zero-coupon bond

$$P = \frac{F}{(1+S(i))^i}$$

$\Rightarrow$ spot rate curve = zero-coupon bond yield curve

better price for level-coupon bonds (given, say, "riskless" US treasury bond to determine $S(i)$):

$$P = \sum_{i=1}^{n} \frac{C}{(1+S(i))^i} + \frac{F}{(1+S(n))^n}$$

, $d(i) := (1+S(i))^{-i}$ called discount factors

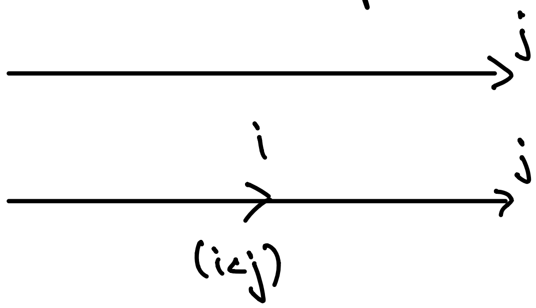
$$P = \sum_{i=1}^{n} d(i)C + d(n)F$$

called "static spread"

risky bonds should be cheaper: replace $(1+S(i))^{-i}$ by $(1+s+S(i))^{-i}$.

Forward Rates:

consider zero-coupon bonds:



$$\longrightarrow FV_j = P(1+S(j))^j$$

$$\longrightarrow FV_i = P(1+S(i))^i$$

$$FV_j = P(1+S(i))^i(1+S(i,j))^{j-i}$$

$S(i,j) :=$ $(j-i)$-period spot rate i periods from now (unknown)

(implied) forward rate $f(i,j)$ = guess for $S(i,j)$ based on

$$(1+S(j))^j = (1+S(i))^i(1+f(i,j))^{j-i}$$

$$\Rightarrow f(i,j) = \left(\frac{(1+S(j))^j}{(1+S(i))^i}\right)^{\frac{1}{j-i}} - 1$$