Maximally entangled qubits
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Alan Huckleberry

The basic vector space of quantum information theory is the q-fold tensor product $V = \mathbb{C}^2 \otimes \cdots \otimes \mathbb{C}^2$ equipped with its standard unitary structure. The state space is the projective space $\mathbb{P}(V)$ equipped with its canonically associated symplectic (Hamiltonian) structure. Physical considerations in this context are almost always invariant under the action of the group $K = SU_2 \times \ldots \times SU_2$ and in many contexts the complexification $G := K^{\mathbb{C}} = SL_2(\mathbb{C}) \times \ldots \times SL_2(\mathbb{C})$ is of relevance.

As a mathematician one learns that, although highly entangled tensors (resp. states) are of physical importance, there is no widely accepted measure of an entanglement function which should be maximized. For the purpose of introducing such a function, our previous work (joint with M. Kús and A. Sawicki and carried much further by Sawicki) calls attention to critical sets of moment maps μ. As an example, the states in the 0-level set of μ maximize a certain variance function which is arguably a first rough measure of entanglement.

The set $\mu^{-1}(0) = X_{KN}$, known as the Kempf-Ness set of invariant theory, contains the usual states which are classically regarded as being highly entangled. However, X_{KM} is a high-dimensional (singular) set which contains a multitude of other states which are equally entangled from the point of view of this first measuring device.

In very interesting work (Sudbury et al), physicists and numerical analysts in Leeds suggested *new* measures of entanglement. They applied numerical (computer) procedures to compute them and algorithms which converge to states which maximize them. These limit states are not the same as those which classically were thought to be maximally entangled. However, they are in the Kempf-Ness set! In a joint project with I. Popanu (Jacobs), for $n \leq 4$ we were able to characterize the "Leeds-states" in terms of the geometric invariant theory of the situation.

Our lecture will first be devoted to the mathematical background of the setting described above. In particular, a concrete description of X_{KN} will be presented. Some ideas on the connection of the geometry of the K- and G-actions and the special Leeds-states will be explained.