Session 6
Sep. 19, 2019

2. Options and Binomial Tree Models

2.1 Options Basics

option = contract depending on future price of some underlying asset
(e.g., stock, usually assumed here)

=> this is a type of "derivative" financial instrument

call option: holder can buy underlying asset for price K at time T

put option: holder can sell underlying asset for price K at time T

option characterized by: • expiration date T
• strike price K

terminology: • payoff = value of option at expiration T
• underlying asset's price S

Ex.: strike price 50 $
suppose at T the stock price is 60 $
↳ call option (buy): payoff = 10 $ (exercise option)
↳ put option (sell): payoff = 0 $ (not exercise option)

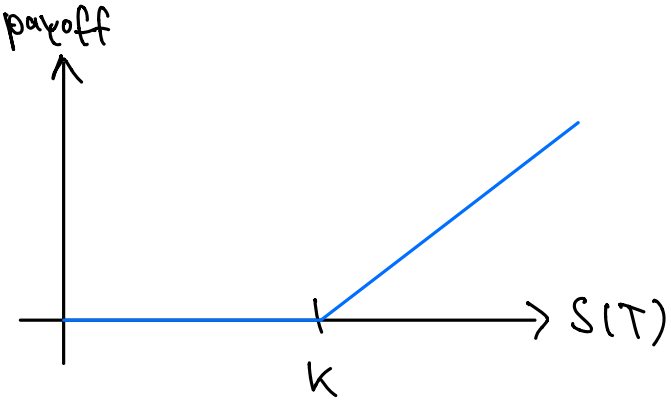
note: profit = payoff − option price

European option: can be exercised only at expiration

American option: can be exercised any time at or before expiration

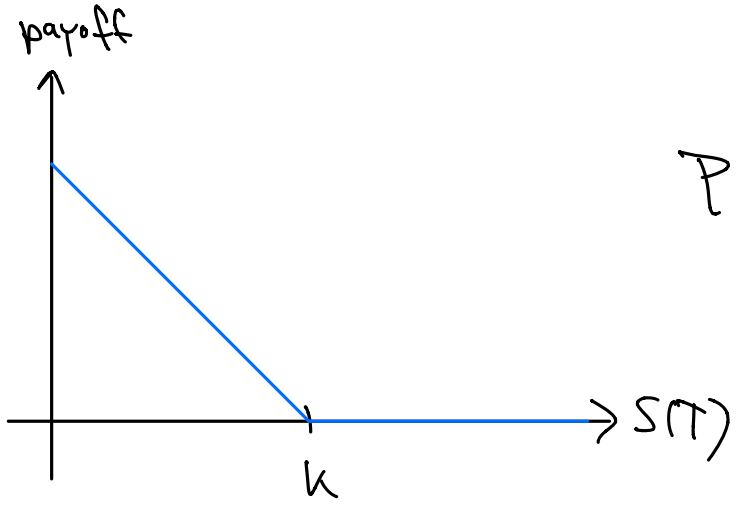
options can be used, e.g., as insurance, speculation etc.

call payoff:



$$C = \max(0, S(T) - K)$$

put payoff:

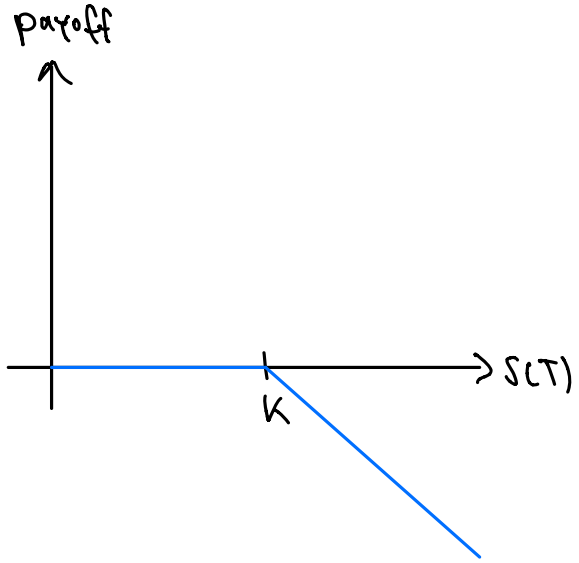


$$P = \max(0, K - S(T))$$

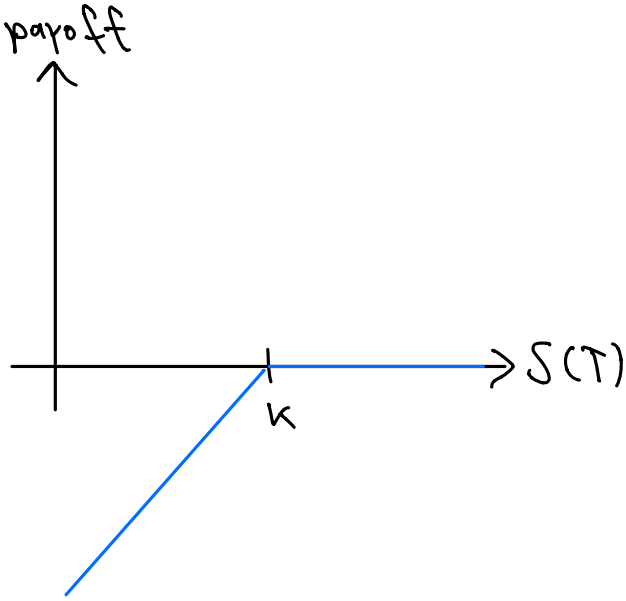
buying option: long position (we usually assume that)

selling option: short position

↳ short a call:



↳ short a put:



Goal (for most of the rest of this class):

What should be price of an option?

Assumptions:

- there is a risk-free market, which we take to be the bond market, with risk-free interest rate r (e.g., US treasury bonds)
- stocks / bonds can be bought and sold unlimitedly and without transaction costs

Uncertainty: $S(T) \longrightarrow$ some probabilistic model

starting point for all pricing models:

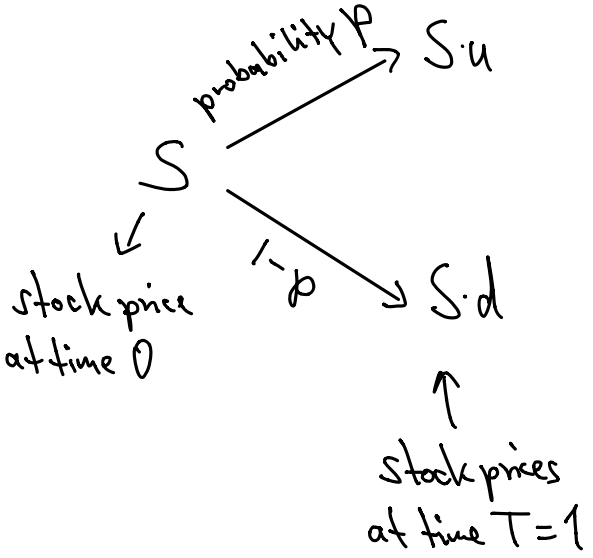
no opportunity for risk-free profit

= no arbitrage assumption

2.2 Binary Model

first we ask this question within a simple binary model

$\longrightarrow$ 2 possibilities for $S(T)$



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Example: $S = 2500\$$, $K = 3000\$$, $r = 0$, call (long)

$p=\frac{1}{2}$ → $4000\$$ → payoff: $1000\$$

$S = 2500\$$

$p=\frac{1}{2}$ → $2000\$$ → payoff: $0\$$

one possibility: set price at $C = \frac{1}{2} \cdot 1000\$ + \frac{1}{2} \cdot 0\$ = 500\$$

then the seller could have the following strategy:

sell option, borrow 2000$, buy one stock (for 2500$)

↳ if $S(T) = 4000\$$ → option will be exercised, sell stock for $K = 3000\$$

⇒ profit: $3000\$ - 2000\$ = 1000\$$

↳ if $S(T) = 2000\$$ → option will not be exercised, sell stock for $2000\$$

⇒ profit: $2000\$ - 2000\$ = 0\$$

⇒ bad option price, since seller has opportunity for risk-free profit

general idea: construct portfolio that mimics option price, called

replicating portfolio