

## 2.5 Convergence Rates

Session 10  
Oct. 9, 2019

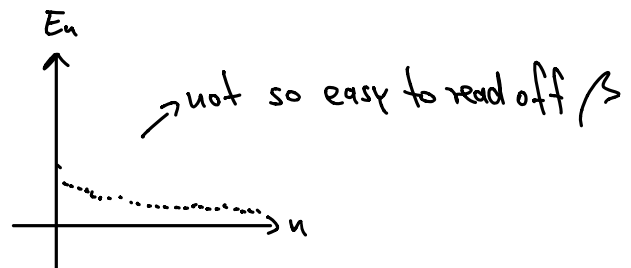
consider sequence  $C_n \xrightarrow{n \rightarrow \infty} C$ , e.g.  $C_n =$  option price for  $n$ -step binomial tree  
(next class:  $\lim_{n \rightarrow \infty} C_n(T=0) =$  Black-Scholes formula)

important: how fast is convergence?

→ usually/often it is a power law:  $E_n = |C_n - C| \approx A \cdot n^{-\beta}$  for large  $n$   
↳ we call  $\beta$  the rate of convergence

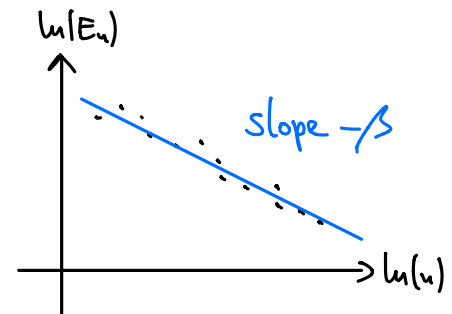
note: if  $C$  is unknown, look at  $|C_n - C_N|$  for some  $N \gg n$

How to read it off from a plot?



⇒ use that  $\ln E_n \approx \ln(A n^{-\beta}) = \ln A - \beta \ln(n)$

⇒ plot  $\ln(E_n)$  against  $\ln(n)$



if we get a straight line with slope  $-\beta$ , we have found the conv. rate

python:  $\log(\log(n, E_n)) \Leftrightarrow \text{plot}(\ln(n), \ln(E_n))$