Calculus and Linear Algebra II

Quiz 2

Instructions:

• Do all the work on this quiz paper.

• Show your work, i.e., write down the steps of your solution cleanly and readable.

• Electronic devices and notes are not allowed.

Name: _________________________________
Problem 1 [7 points]

(a) Let $f(x)$ be infinitely often differentiable at 0. Write down the definition of the (infinite) Taylor series of $f$ around 0.

(b) Compute the Taylor series of $f(x) = \sin(x)e^{2x}$ up to quadratic order (i.e., including the $x^2$ term).

(c) Does the integral $\int_{-1}^{1} x^{-2} \, dx$ exist as an improper integral (or a sum of improper integrals)? If yes, what is its value? If no, explain why.
Problem 1 (extra space)
Problem 2 [8 points]

(a) For $f(x, y) = x^2y^2 + \ln(x) \cos(y)$, compute the partial derivatives $\partial_x f$ and $\partial_y f$.

(b) Define what it means for a function $f : \mathbb{R}^n \to \mathbb{R}$ to be differentiable at $\vec{a} \in \mathbb{R}^n$.

(c) Suppose we have three functions $f : \mathbb{R}^n \to \mathbb{R}$, $g : \mathbb{R}^n \to \mathbb{R}$, and $h : \mathbb{R}^n \to \mathbb{R}$ given. Simplify $\nabla (fgh)$ with the product rule.

(d) What is the differential $df$ of $f(x, y) = \sqrt{x^2 + y^2}$?
Problem 2 (extra space)