

# Calculus and Linear Algebra II

## Quiz 2

**Instructions:**

- Do all the work on this quiz paper.
- Show your work, i.e., write down the steps of your solution cleanly and readable.
- Electronic devices and notes are not allowed.

**Name:** \_\_\_\_\_



**Problem 1 [7 points]**

- (a) Let  $f(x)$  be infinitely often differentiable at 0. Write down the definition of the (infinite) Taylor series of  $f$  around 0.
- (b) Compute the Taylor series of  $f(x) = \sin(x)e^{2x}$  up to quadratic order (i.e., including the  $x^2$  term).
- (c) Does the integral  $\int_{-1}^1 x^{-2} dx$  exist as an improper integral (or a sum of improper integrals)? If yes, what is its value? If no, explain why.

**Problem 1 (extra space)**

**Problem 2 [8 points]**

(a) For  $f(x, y) = x^2y^2 + \ln(x) \cos(y)$ , compute the partial derivatives  $\partial_x f$  and  $\partial_y f$ .

(b) Define what it means for a function  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  to be differentiable at  $\vec{a} \in \mathbb{R}^n$ .

(c) Suppose we have three functions  $f : \mathbb{R}^n \rightarrow \mathbb{R}$ ,  $g : \mathbb{R}^n \rightarrow \mathbb{R}$ , and  $h : \mathbb{R}^n \rightarrow \mathbb{R}$  given. Simplify  $\vec{\nabla}(fgh)$  with the product rule.

(d) What is the differential  $df$  of  $f(x, y) = \sqrt{x^2 + y^2}$ ?

**Problem 2 (extra space)**