Jacobs University | March 25, 2020
Spring 2020

Calculus and Linear Algebra II

Quiz 4

Instructions:

- Do all the work on this quiz paper.
- Show your work, i.e., write down the steps of your solution cleanly and readable.
- Electronic devices and notes are not allowed.

Name: ______________________________

Problem 1 [8 points]

Let $f(x, y) = 1 + xy$. Using the method of Lagrange multipliers, find the critical points of f on the unit circle, i.e., under the constraint $G(x, y) = x^2 + y^2 - 1 = 0$. (You do not need to check whether they are maxima or minima or neither, but you can try to find out if time is left.)

Problem 1 (extra space)

Problem 2 [7 points]

(a) Let

$$\vec{f} : \mathbb{R}^2 \to \mathbb{R}^3, (x, y) \mapsto \vec{f}(x, y) = \begin{pmatrix} x^2 y \\ \sin(x) \\ e^{x+y} \end{pmatrix}.$$

Determine the Jacobian matrix of $\vec{f}$.

(b) Let

$$\vec{f} : \mathbb{R}^3 \to \mathbb{R}^3, (x, y, z) \mapsto \vec{f}(x, y, z) = \begin{pmatrix} x^2 yz \\ x^2 + y^2 + z^2 \\ x^2 + y^2 + z^2 \end{pmatrix}.$$

Determine $\operatorname{curl} \vec{f}$. Then, compute $\operatorname{div}(\operatorname{curl} \vec{f})$.

Problem 2 (extra space)