Moodle Exercise Set 2

Calculus and Linear Algebra II

Spring 2020

1. What is the n-th term in the Taylor series of $f(x) = \cos(x)$ centred at $b = \pi$?
   A. $(-1)^{n+1}\frac{(x-\pi)^{2n}}{(2n)!}$
   B. $(-1)^{n+1}\frac{x^{2n}}{(2n)!}$
   C. $(-1)^{n+1}\frac{(x-\pi)^{2n+1}}{(2n+1)!}$
   D. $(-1)^{n+1}\frac{x^{2n+1}}{(2n+1)!}$
2. What is the n-th term in the Taylor series of $f(x) = e^x$ centred at $b = 3$?
   A. $\frac{e^3}{n!}(x-3)^n$
   B. $\frac{e^3 x^n}{n!}$
   C. $\frac{x^n}{n!}$
   D. $\frac{(x-3)^n}{n!}$
3. What are the first three non-zero terms in the Taylor series of $f(x) = e^{-x^2}\cos(x)$ centred at $b = 0$?
   A. $1,\ -\frac{3}{2}x^2,\ \frac{25}{24}x^4$
   B. $1,\ -x^2,\ \frac{1}{2}x^4$
   C. $1,\ -\frac{1}{2}x^2,\ \frac{1}{24}x^4$
   D. $1,\ \frac{x^4}{2},\ \frac{x^8}{48}$
4. The hyperbolic sine is defined by $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$. What is the n-th term in the Taylor expansion of $\sinh(x)$ around $b = 0$?
   A. $\frac{x^{2n+1}}{(2n+1)!}$
   B. $\frac{x^{2n}}{(2n)!}$
   C. $\frac{(-1)^n}{(2n+1)!}x^{2n+1}$
   D. $\frac{(-1)^n}{(2n)!}x^{2n}$
5. The hyperbolic cosine is defined by $\cosh(x) = \frac{1}{2}(e^x + e^{-x})$. What is the n-th term in the Taylor expansion of $\cosh(x)$ around $b = 0$?
   A. $\frac{x^{2n+1}}{(2n+1)!}$
   B. $\frac{x^{2n}}{(2n)!}$
   C. $\frac{(-1)^n}{(2n+1)!}x^{2n+1}$
   D. $\frac{(-1)^n}{(2n)!}x^{2n}$
6. What is the value of $\sum_{k=1}^{\infty} \frac{k^2}{k!}$? (This exercise is slightly trickier than the rest. Hint: try to rewrite the sum as a Taylor series you are familiar with.)
   A. e

B. $2e$

C. $(e+1)(e-1)$

D. e^2

7. Let F_N be the N-th order Taylor polynomial of $f(x) = \sqrt[3]{x}$ at $b = 8$. How accurate is the approximation by F_2 when $7 \leq x \leq 9$?

   A. The approximation is accurate to ≈ 0.4

   B. The approximation is accurate to ≈ 0.04

   C. The approximation is accurate to ≈ 0.004

   D. The approximation is accurate to ≈ 0.0004

8. Let F_N be the N-th order Taylor polynomial of e^x centred at $b = 0$. What is the smallest value of N for which $F_N(1)$ approximates e to two decimal places?

   A. $N = 3$

   B. $N = 4$

   C. $N = 5$

   D. $N = 6$

9. Use Newton's method on $x^2 - a = 0$. What is x_{n+1}?

   A. $\frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$

   B. $x_n - \frac{2x_n}{x_n^2 - a}$

   C. $x_n + \frac{a}{x_n}$

   D. $x_n - \frac{a}{x_n}$

10. Use Newton's method on $x^3 + x + 3 = 0$ to compute x_2 when the initial approximation is $x_1 = -1$.

    A. $-5/4$

    B. $-3/2$

    C. 1

    D. $3/4$

11. Use Newton's method on $x^3 + 2x - 4 = 0$ to compute x_2 when the initial approximation is $x_1 = 1$.

    A. $6/5$

    B. -1

    C. $4/5$

    D. $1/5$

12. Use Newton's method on $x^5 - x - 1 = 0$ to compute x_2 when the initial approximation is $x_1 = 1$.

    A. $5/4$

    B. $1/4$

    C. $3/4$

    D. 1

13. Consider the integral $\int_{-\infty}^{-1} \frac{1}{\sqrt{2-x}} dx$. Does the integral converge? If yes, evaluate the integral.

    A. The integral converges to 0

    B. The integral converges to 1

    C. The integral converges to $1/\sqrt{2}$

    D. The integral diverges.

14. Consider the integral $\int_{-\infty}^{\infty} xe^{-x^2} dx$. Does the integral converge? If yes, evaluate the integral.

A. The integral converges to 0

B. The integral converges to e

C. The integral converges to $1/e$

D. The integral diverges

15. Consider the integral $\int_1^\infty \frac{\ln(x)}{x} dx$. Does the integral converge? If yes, evaluate the integral.

A. The integral converges to 0

B. The integral converges to $\frac{\ln(1+e)}{2}$

C. The integral converges to e

D. The integral diverges

16. Consider $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$. Can we use the integral test here? If yes, use the integral test to determine whether the series converges.

A. The integral test cannot be used.

B. The integral test can be used and the series converges.

C. The integral test can be used and the series diverges.

D. The integral test can be used but is inconclusive.

17. Consider $\sum_{n=1}^{\infty} ne^{-n}$. Can we use the integral test here? If yes, use the integral test to determine whether the series converges.

A. The integral test cannot be used.

B. The integral test can be used and the series converges.

C. The integral test can be used and the series diverges.

D. The integral test can be used but is inconclusive.

18. Consider $1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \cdots$ Can we use the integral test here? If yes, use the integral test to determine whether the series converges.

A. The integral test cannot be used.

B. The integral test can be used and the series converges.

C. The integral test can be used and the series diverges.

D. The integral test can be used but is inconclusive.

19. Consider $\sum_{n=1}^{\infty} \frac{1}{n^2+4}$. Can we use the integral test here? If yes, use the integral test to determine whether the series converges.

A. The integral test cannot be used.

B. The integral test can be used and the series converges.

C. The integral test can be used and the series diverges.

D. The integral test can be used but is inconclusive.

20. Consider $\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$. Use the integral test to determine if the series is convergent or divergent. If the corresponding integral converges, evaluate the integral.

A. The integral test cannot be used.

B. The integral test can be used and the series converges.

C. The integral test can be used and the series diverges.

D. The integral test can be used but is inconclusive.