Moodle Exercise Set 3

Calculus and Linear Algebra II

Spring 2020

1. What is the integral $\int_{-1}^{0} \frac{e^{\frac{1}{x}}}{x^3}\ dx$?

2. What is the integral $\int_{0}^{2} x^2 \ln(x)\ dx$?

3. Let $f(x,y) = \frac{x-y}{x+y}$. What is $\frac{\partial f}{\partial x}$?

4. Let $z(x,y) = x^2 y^3$ and $x(s,t) = s\cos(t)$ and $y(s,t) = s\sin(t)$. Compute the partials $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

5. Let $z(x,y) = e^x \cos(y)$ and $x(s,t) = st$ and $y(s,t) = \sqrt{s^2 + t^2}$. Compute the partials $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

6. Let $f(x,y,z) = \ln(x + 2y + 3z)$. What is $\frac{\partial f}{\partial z}$?

7. The total resistance R produced by three conductors with resistances R_1, R_2, R_3 connected in a parallel circuit is given by the formula

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

What is $\frac{\partial R}{\partial R_1}$?

8. The gas law for a fixed mass m of an ideal gas at absolute temperate T, pressure P, and volume V is $PV = mRT$, where R is a constant. What is the value of $T \cdot \frac{\partial P}{\partial T} \cdot \frac{\partial V}{\partial T}$?

9. The temperature (in Celsius) at a point (x,y,z) is given by $T(x,y,z) = 200 e^{-x^2 - 3y^2 - 9z^2}$. What is the rate of change (i.e., the gradient) of temperature at $(2,-1,2)$?

10. Suppose that over a region of space the electric potential V is given by $V(x,y,z) = 5x^2 - 3xy + xyz$. What is the rate of change (i.e., the gradient) of the potential at $(3,4,5)$?

11. What is the gradient of $f(x,y) = \frac{y^2}{x}$ at $(2,4)$?

12. What is the gradient of $f(x,y,z) = \frac{xy + yz + xz}{\sqrt{x^2 + y^2 + z^2}}$ at $(3,6,-2)$?

13. What is the gradient of $f(x,y,z) = xe^y \cos(z) - z - 1$ at $(1,0,0)$?

14. What is the equation of the tangent place to the surface defined by $f(x,y) = y\ln(x)$ at the point $(1,4)$?

15. What is the equation of the tangent plane to the surface defined by $x^2 - 2y^2 + z^2 + yz = 2$ at $(2,1,-1)$?

16. What is the differential of $f(x,y) = x^3 \ln(y^2)$?

17. What is the differential of $R(\alpha, \beta, \gamma) = \alpha\beta^2 \cos(\gamma)$?