

Moodle Exercise Set 5

Calculus and Linear Algebra II

Spring 2020

1. Let $f(x, y, z) = xe^{y/z}$ and let $x = t^2$, $y = (1 - t)$ and $z = 1 + 2t$. What is $\frac{df}{dt}$?
2. The temperature at a point (x, y) is given by a differentiable function $T(x, y)$. A small insect crawls so that its position after t seconds is given by $x(t) = \sqrt{1+t}$ and $y(t) = 2 + \frac{t}{3}$. The temperature function satisfies $\partial_x T(2, 3) = 4$ and $\partial_y T(2, 3) = 3$. How fast is the temperature rising on the insect's path after 3 seconds?
3. The length l , width w , and height h of a box change with time. At a certain time, the dimensions are $l = 1\text{m}$, $w = h = 2\text{m}$. Moreover l and w are increasing at a rate of $2\text{m} \cdot \text{s}^{-1}$. and h is decreasing at a rate of $3\text{m} \cdot \text{s}^{-1}$. At that instant find the rate at which the surface area of the box is changing.
4. What are all the critical points of the function $f(x, y) = e^x \cos(y)$.
5. What are all the critical points of the function $f(x, y) = (x^2 + y^2)e^{y^2 - x^2}$?
6. What are all the critical points of the function $f(x, y) = (1 + xy)(x + y)$?
7. How many critical points does $f(x, y) = y^2 - 2y \cos(x)$ have in the strip $-1 \leq x \leq 7$?
8. How many critical points does $f(x, y) = x^2 + 4y^2 - 4xy + 2$ have?
9. Consider the function $f(x, y) = 9 - 2x + 4y - x^2 - 4y^2$. First identify the critical points and then plot the function. What value does the function take at the local maxima?
10. Consider the function $f(x, y) = x^3 - 12xy + 8y^3$. First identify the critical points and then plot the function. What value does the function take at the local minima?